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Classical Interpretation of Quantum Mechanical Concepts

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Abstract

All of the mathematical formulations of quantum mechanics describe the interaction between radiation and atoms, but we show here that only Einstein has been able to establish a solid logical connection with classical theory. His methods are used to interpret the other mathematical models by requiring that observables be described in a physically-defined coordinate system and that there be strict adherence to the conservation of momentum. The properties of momentum exchange are included in a quantum oscillator, or clock, by replacing the non-relativistic Hamiltonian model with a relativistically correct Lagrangian model. Equal contributions are assigned to photon, electron, and nucleus; and field boundaries are defined that coincide with the electron shells. The fields of the particles superpose linearly, but otherwise their physical integrity is maintained throughout. The Lagrangian model accounts for the 720 degree rotation of a wave function as an initial 360 degree electromagnetic wave cycle (photon absorption) followed by a second wave cycle (photon emission), yielding two wave cycles to correspond with one complete electron cycle. The abstract space of quantum mechanics is replaced by a real classical space.

Introduction

The three quantum mechanical formulations

Feynman referred to the three formulations of quantum mechanics; matrix mechanics, wave mechanics, and path integrals; as different "points of view" of the same thing [1]. He was of course talking about the mathematics, but he could also have been referring to a physical relationship *since they all describe the same natural phenomenon*; the emission and absorption of radiation by an atom. Due to the physical continuity that exists there must be a way to connect the various mathematical models. Our inability to transform directly from one mathematical formalism to another means that certain characteristics of the wave function are specific to the mathematics. Interpretation is dependent upon certain facts, facts that are supported by experiment to be sure, but not a complete rendering of the facts since each interpretation or model of quantum mechanics is supported by a different set of experiments. As Schrödinger pointed out, the differences between matrix and wave mechanics are many [2], "starting-points, presentations, methods, and in fact the whole mathematical apparatus, seem fundamentally different." No model of quantum mechanics is able to give a complete description because each one addresses a different aspect of the physical phenomenon..

The three-body model

Quantum mechanics is currently identified with the mathematical models that describe it. All are derived from the same physical origin, the absorption and emission of radiation, but they describe it in very different ways. Our purpose here is not to discover why quantum mechanics cannot be de-scribed by a single model, rather it is to determine why the derivations have very distinct appearances. The current physical model, proposed originally by Bohr and perpetuated by all subsequent models consists of two bodies, a circulating negatively charged electron bound to the potential of a much more massive positively charged nucleus. This is referred to as the Hamiltonian model, H=T+V, where T is the energy of the electron and V is its potential. Add energy to the atom and the potential of the electron increases, subtract energy and the potential is released in the form of a photon.

Suppose instead that the photon, which embodies energy and has its own physical properties, is created as an independent entity when energy is absorbed; then quantum mechanics refers to not two, but three bodies. This is a more complicated problem than the classically inspired model of a negatively charged particle orbiting a positively charged potential since it presumes that the three field sources are loosely bound within a conservative, or frictionless system, that they are free to interact

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with each other, and that each of the three particles influences the atomic system independently. For the related case of three particles in a gravitational system no general closed form solution is possible [3]. Gravitationally bound three-body systems result in chaos for nearly all initial conditions. It should not be surprising therefore that a physical system consisting of three electromagnetic field sources; electron, photon, and nucleus; also has an indeterminate outcome. To obtain the equations of motion for an electromagnetic three-body problem when the only knowledge available about the particles is their field properties, we need to obtain a series of partial solutions by superposing particles two at a time. It will be assumed that an exact solution is not possible for the dynamic evolution of a three body system so all solutions are considered approximations.

The three-body model of atomic structure may be described formally by introducing a wave-like, physically independent field source ε, the localized photon, into our description of excited atomic states. The modified Hamiltonian is now given by,

$$H = T_e + \varepsilon + V_n \qquad 1)$$

where Te refers to an electron's field, ε represents the field of a "captured" photon, and Vn represents the positive field of the nucleus. The Lagrangian is similarly given by,

$$L = T_e + \varepsilon - V_n \qquad 2)$$

The equations 1) and 2) contain the essence of quantum mechanics as a three-body conservative system in real space, as opposed to nonrelativistic descriptions in abstract space. The equations revert to their classical two-body form when the influence of ε is negligible. Each of the three field sources (or particles), possesses a unique vector field; that is, a well-defined field geometry, while the plus and minus signs indicate that the superposition of their fields is linear.

Mathematical formulations

Bohr atom

The simplest physical model approximating atomic structure is the Bohr atom. It consists of electron shells arranged concentrically around the nucleus. The electron occupies allowed "orbitals" and it emits electromagnetic energy in discrete bundles, or quanta, according to the relation,

$$E_2 - E_1 = h\nu \qquad 3)$$

for changes between an excited state , and a ground state . The stationary states are determined by the angular momentum of the revolving electron where n is the principal quantum number.

$$m_e\, vr = n\hbar, \quad wheren=1,2,3$$

Changes in state occur according to the Hamiltonian model, H=T+V, when an electron with energy T increases its potential energy V sufficiently to "jump" to the next higher energy level. Because the Bohr model describes the hydrogen atom as an electron "orbiting" the nucleus, or proton, in a bound state, but does not explicitly require the presence of photons Bohr did not believe in the photon's existence for many years [4]. Because the model is unable to explain more detailed interactions such as the emission and absorption of radiation, it soon became evident that an improved model was necessary.

Matrix mechanics

Improvements to the Bohr model came with the publication of a series of papers concerning the interaction of electromagnetic radiation with matter. The first was Einstein's seminal paper "The quantum theory of radiation" [5]. Making use of results obtained in an earlier paper on Brownian motion he was able to show that black body radiation is the result of a dynamic equilibrium between classical and quantum mechanical laws. Molecules that absorb thermal energy according to the classical laws of statistical thermodynamics then emit a time-averaged equal amount of energy quantum mechanically according to the statistics of the Planck radiation law. Induced absorption occurs continuously due to random inputs of momentum from thermal collisions and radiation, while induced and spontaneous emission occurs discretely according to the Bohr frequency rule 3) and is directed along an infinitesimal solid angle consistent with a photon's momentum E/c. Dynamic equilibrium between classical and quantum statistics is ensured by the conservation of momentum.

The momentum of light was realized long before Maxwell's equations came into existence when it was noticed that a comet's tail always extends away from the sun [6]. There has also been significant progress in recent years in the use of momentum in the astronomical sciences, molecular ma-nipulation, optical tweezers, and laser cooling. In spite of our vastly improved knowledge about the mechanical properties of the photon these technological advances have not translated into an improved theoretical understanding in any of the formulations of quantum mechanics. Einstein's remains the only theory of radiation that depends upon the influence of momentum theoretically to describe absorption and emission.

The molecular equilibrium in black body radiation that Einstein described is similar in many respects to the emission and absorption of energy observed in dispersion phenomena and spectral lines. Dispersion is the continuous change in the angle of refraction of different frequencies of light by a prism or other medium. Although light disperses continuously across the entire spectrum, at certain specific frequencies characteristic of the medium, it is completely absorbed forming lines. When Bohr introduced his theory of electron orbitals he immediately recognized the possibility that the discrete lines of atomic spectra are related to the discrete lines in dispersion phenomena [7]. Subsequently there were various unsuccessful attempts to explain the discrete and continuous properties of dispersion [8, 10,13]. Ladenburg was finally able to combine the Bohr frequency condition with the Einstein A and B coefficients to form a statistical balance between quantum mechanical and classical energy exchange by equating two theoretical expressions, the energy absorbed/emitted by N classical resonators and the energy absorbed/emitted by N' quantum atoms [9,10,13]. Although Ladenburg did not take momentum into account in his analysis, he was able to preserve the statistical balance of energy at the molecular level thereby differentiating between classical and quantum mechanical properties, an important aspect of Einstein's derivation. Four years later Kramers reinterpreted Ladenburg's results by using the Bohr model of the atom as a multiply periodic system of virtual oscillators [11,12,13]. In that model a quantum mechanical variable X is described with a classical Fourier series, where A(n, n- τ) is the quantum analog of the classical amplitude, n indicates the electron orbital number, and τ assumes integral values to denote positive or negative transitions [12,13].

$$X = \sum_{\tau} A(n, n-\tau)\exp[2i\pi\nu(n, n-\tau)t], \tau = \bar{+}1, \bar{+}2, \ldots \qquad (4)$$

The Bohr-Kramer method distanced itself from that of Einstein in an important way. Einstein used the conservation of momentum as a guiding principle to compare classical properties observed in laboratory coordinates with quantum mechanical properties observed in particle coordinates. The continuous

properties of matter described in laboratory coordinates are thereby distinguished from discrete properties described by particle coordinates. In the interpretation by Fourier series 4) all properties of matter-radiation interactions are described relative to laboratory coordinates. Discrete changes in energy due to "electron jumps" are described by pairs of states, while momentum, which is transmitted at single points in space and time, is ignored. In other words, the concept of photon momentum, an intrinsic property whose displacement in time is directional, is replaced by a wave model that is isotropic and treats emission and absorption as continuous processes that are reversible in time.

Once Kramer had reinterpreted Einstein's quantum theory of radiation Heisenberg was able to use it to formulate a theory of quantum mechanics by using virtual harmonic oscillators to reconcile the continuity of radiation fields with the discrete energy states of an atom [12,13]. The complex sets of mathematical rules of the type 4), that he used to describe the frequencies and intensities of spectral lines, are expressed in the form of a matrix:

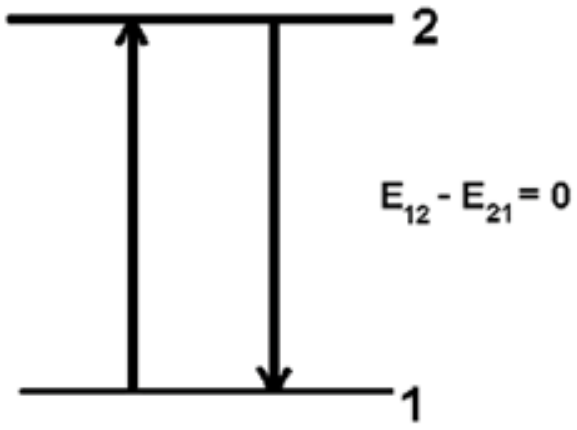
$$\sum_k (p_{nk}q_{km} - q_{nk}p_{km}) = \begin{matrix} ihforn = m \\ 0\, forn \neq m \end{matrix} \qquad (5)$$

The matrix products do not always commute as they would in classical theory. Whenn=mthe elements are diagonal and the value of the equation is equal to iħ. For the non-diagonal elements,n≠m, its value is zero.

The electron oscillator

After the modifications by Ladenburg and Heisenberg Einstein's ideas are no longer recognizable. This despite the fact that the reinterpretations describe the same physical phenomenon, the interaction between matter and radiation. The overwhelming difference between them is their complete disregard for momentum and the specification of an appropriate coordinate system. Einstein used coordinates fixed with respect to a molecule to derive his A and B coefficients governing the quantum mechanical absorption and emission of energy [13]. He next defined a molecule's motion in response to thermal forces by using a coordinate system fixed with respect to laboratory coordinates. This allowed him to describe the dynamic equilibrium between opposing forces on molecules due to the conservation of momentum and also maintain a clear separation between classical and quantum observables. In contrast, the Bohr-Kramers method describes all observables, discrete and continuous, in laboratory coordinates. There was no need as far as Heisenberg was concerned to describe the discrete spectral lines due to atomic orbitals and the continuous observables due to dispersion phenomena differently, concluding that [14], "Quantum mechanics [is] founded exclusively upon relationships between quantities which are in principle observable."

Dispersion phenomena are observed and measured in laboratory coordinates. They are given by off-diagonal elements of matricesn≠mwhere elements above the diagonal refer to energy absorption and elements below the diagonal refer to energy emission. Because they are resonances they do not induce significant amounts of radiation. The energy of an absorption offsets the energy of an emission except for a difference in phase so a value of zero is obtained for equation 4). On the other hand, the diagonal elements of matrices forn=mare real eigenvalues representing ground state energy levels.



Absorption results in stimulation to a higher orbital and the subsequent emission of a photon upon decay. As Dirac explained [15] "Heisenberg assumed that one should consider the whole set together [the matrix] as corresponding to one of the dynamical variables of the Newtonian theory. These dynamical variables are of course the coordinates of the particles, or the velocities, or momenta." However, Einstein's methods require that classical and quantum mechanical observables be evaluated with distinct sets of coordinates. The matrices represent observables measured in laboratory coordinates and using them to describe the dynamical variables of atomic structure requires a transformation of coordinates.

A transformation of coordinates is possible by introducing the idea of electron oscillator. The figure on the left shows how induced absorption and emission occur with respect to an electron oscillator, where arrows refer to electron transitions. Due to absorption the electron is raised from the ground state to an excited state . When the electron returns to it completes a cycle of the electron oscillator and a photon is irreversibly emitted. Each arrow represents one-half cycle of the electron oscillator and one full cycle of an electromagnetic wave.

Wave mechanics

The concept of electron oscillator may be used to describe the rotation of the wave function of any half-integer spin particle [16]. According to the Schrödinger wave equation radiation occurs as a single process that evolves symmetrically in time. The wave function ψ performs two complete rotations, or a total of 720 degrees, before returning to its original state. If we apply Einstein's methods excitation consists of the rotation of an electron's wave function through 2π radians during the *absorption* of one complete cycle of an electromagnetic wave and momentum is directed inward. Decay corresponds to a second rotation of 2π radians during the *emission* of a complete wave cycle, and momentum is directed outward. We interpret rotations of the wave function not as rotations in *abstract space,* but as changes in phase of electromagnetic fields from 0 to 2π in *real space.* In other words, a complete electron cycle, excitation and decay, consists of two wave function rotations, or 4π radians and two cycles of an electromagnetic wave. The completed 720 degree wave function rotation consists of one cycle of an electron oscillator and two cycles of a wave. The dual wave-particle nature of the photon is thereby realized as a physical transformation. Thus changes in state can be viewed as excitations and decay,

wave function rotations, cycles of a wave, or superpositions of fields depending upon the physical model.

The transfer of momentum to a molecule by photons was predicted theoretically by Einstein and verified by experiments of many types. It has also been verified microscopically by recent experiments with ultracold three-level artificial atoms which support the idea that momentum is a necessary parameter for the description of emission processes [17]. In the quantum Zeno effect frequent measurements can arrest the progress of a "quantum jump". The measurements are equivalent to momentum transfers once again experimentally confirming Einstein's theory that photon momentum needs to be included in theories of the absorption and emission of radiation. An incoming photon transfers a momentum(+E)⁄cto an atom in the ground state together with a superposition of fields. As it exits the superposition state it transfers recoil momentum(-E)⁄cto the atom and is expelled. Because induced absorption and emission momenta are directed in opposite directions a discontinuity occurs during the cycling of the electron oscillator. However, in the wave mechanical view momentum exchange does not occur during the absorption and emission of energy, causing energy exchange to be symmetrical and time to be reversible. Nowhere else in nature is it possible to discount the influence of momentum. This means that the Schrödinger wave equation is an incomplete description of radiation processes because it does not include discontinuities due to momentum exchange.

Lagrangian quantum mechanics

We wish to obtain a wave equation that includes photons, in other words, a relativistic equation. Dirac gave reasons why a relativistic wave equation would be desirable [18]. "There is an alternative formulation for classical dynamics, provided by the Lagrangian. This requires one to work in terms of coordinates and velocities instead of coordinates and momenta. The two formulations are, of course closely related, but there are reasons for believing that the Lagrangian one is the more fundamental. In the first place the Lagrangian method allows one to collect together all the equations and express them as the stationary property of a certain action function. (This action function is just the time-integral of the Lagrangian.) There is no corresponding action principle in terms of the coordinates and momenta of the Hamiltonian theory. Secondly the Lagrangian method can easily be expressed relativistically, on account of the action function being a relativistic invariant; while the Hamiltonian method is essentially non-relativistic in form, since it marks out a particular time variable as the canonical conjugate of the Hamiltonian function. For these reasons it would seem desirable to take up the question of what corresponds in the quantum theory to the Lagrangian method of the classical theory."

Dirac proceeded by saying that, "We ought to consider the classical Lagrangian not as a function of the coordinates and velocities but rather as a function of the coordinates at time t and the coordinates at time t+dt"; a suggestion which gave Feynman the idea to pursue a path integral formulation of quantum mechanics. In other words, rather than specify photon emission as an event that occurs at a particular point in time as in nonrelativistic theory, Dirac seeks compatibility with relativity theory by calculating change in action over a space-time interval between two points in time. We continue with his initiative by letting the coordinates at time t and at time t+dt denote electron shells corresponding to the states and respectively. Next, "We introduce at each point of space-time a Lagrangian density, which must be a function of the coordinates and their first derivatives with respect to x,y,z, and t corresponding to the Lagrangian in particle theory being a function of coordinates and velocities. The integral of the Lagrangian density over any (four-dimensional) region of space-time must then be stationary for all small variations of the coordinates inside the region, provided the coordinates on the boundary remain invariant." Emission initiates from the steady statewith coordinates $R_2 = (x_2,y_2,z_2)$ and time t_2 and it finalizes at the steady state with coordinates $R_1 = (x_1,y_1,z_1)$ and time t_1; where R_2 and R_1 denote electron shells. The Lagrangian density within the four-dimensional space-time region bounded by the electron shells is a function of the coordinates and their first derivatives $L(\phi_i,\phi_{i,\mu})$. The conditions are satisfied by an action integral of the Lagrangian density.

$$S[\phi_i(t)] = \int_{R_2}^{R_1}\int_{t_2}^{t_1} L(\phi_i\phi_{i,u})d^3Xdt = h \qquad (6)$$

The action is a functional, a function of the values of coordinates on the discrete boundaries of the space-time surfaces R2 and R1 which are in turn functions of the continuous space-time variables of the fields within the surface. The Lagrangian density is "stationary for all small variations of the coordinates inside the region provided the coordinates on the boundary remain invariant". The photon is represented as a four-dimensional localization of field, with momentum exchange occurring upon contact with the electron shells R2 and R1.

Conclusion

The mathematical models of quantum mechanics; wave mechanics, matrix mechanics, and the path integral formulation; do not explicitly include the photon. They are nonrelativistic, employing the Hamiltonian model with energy as a potential, which causes the different models to emphasize only two of the three participants in atomic structure, electron and nucleus. Thus each mathematical model is an attempt to visualize one aspect of quantum mechanics as a partial view of the whole. By applying the classical Lagrangian to a radiating atomic system we are able to give a complete interpretation of energy exchange with photon emission visualized as a four-dimensional localization of fields. The Lagrangian method is more fundamental than other formulations of quantum mechanics allowing it to be applied universally in nature. In fact it can be used to show a relationship between photons and galaxies by comparing electromagnetic and gravitational energy exchanges [19].

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