



The Role of the Structure of Matter in its Dynamics and Evolution

Somsikov Vyacheslav Mikhailovich

IETP AI-Farabi Kazakh National University, Almaty, Kazakhstan

Abstract

The article is devoted to the study of the role of the structure of the body in its dynamics and evolution. A system of potentially interacting material points was used as a body model. To study the role of body structures, the equation of motion of the system was used. It is shown how it is derived from the condition of invariance of the total energy of the system. This equation takes into account that the work of external forces goes not only to change the energy of motion of the system, but also to change its internal energy. As a result, it was found that taking into account the structural nature of bodies opens up the possibility of describing the processes of their evolution within the framework of the fundamental laws of physics. Based on the condition of infinite divisibility of matter, which follows from the equation of motion of the system, the general principles of organizing the structures of matter at all its hierarchical levels are considered. Examples are given that demonstrate the importance of taking into account the role of the structure of bodies for the development of physics.

Introduction

Classical mechanics lies at the foundations of modern physics. It is built for a model of bodies in the form of a material point (MP) or a collection of them. In accordance with this model of the body, the formalisms of classical mechanics were built under the conditions of holonomy of constraints and the potentiality of collective forces [1-3]. But in real nature there are no structureless bodies. Therefore, it is obvious that such a simplification of the body model inevitably leads to limitations of theories. As it turned out, one of them is the reversibility of the motion of bodies, which contradicts the second law of thermodynamics [4]. The impossibility of constructing an evolutionary picture of the world within the framework of the laws of physics was one of the consequences of these limitations [5,6].

Evolution, along with movement and interaction, is an integral property of matter. Although evolution includes movement and interactions, it is not reducible to them, since, in addition, it contains the processes of birth, development and decay of systems determined by the properties of the system and external factors. The difficulties in studying evolution are due to the following reasons [7].

Firstly, the processes of evolution are dissipative and irreversible, while classical mechanics is reversible. That is, the internal dynamics of the elements of bodies, described by thermodynamics, and the movement of bodies in space, described by classical mechanics, do not agree with each other.

Secondly, to describe evolution it is

necessary to take into account the openness and nonequilibrium of systems. Indeed, evolution is associated with dissipative processes, with nonlinear phase transitions, bifurcation processes, which lead to the emergence of new structures. These processes are possible only in open nonequilibrium dynamic systems (ONDS). There are no corresponding theories to study them yet.

Thirdly, the processes of evolution are universal for all levels of matter. Therefore, to describe evolution, the unity of physics is required, and not its fragmentation that occurs today.

Attempts to overcome the contradictions between reversibility in classical mechanics and the second law of thermodynamics originate from the works of Boltzmann [4]. Since classical mechanics excludes the possibility of describing evolution, in order to explain the mechanism of irreversibility it was necessary to rely on probabilistic laws [4]. The basis of the existing explanation of irreversibility lies in the Lyapunov exponential instability of Hamiltonian systems and hypotheses about the existence of random external fluctuations. But such a solution to the problem, although it does not contradict both theory and observations, is not consistent with the principle of causality [8]. This casts doubt on the possibility of the existence of a deterministic irreversibility mechanism (DIM) within the framework of existing theories. Therefore, the search for DIM continued [9, 10]. To go beyond this framework, we first studied the role of the structure of bodies in their dynamics by

Correspondence

Somsikov Vyacheslav Mikhailovich

IETP AI-Farabi Kazakh National University,
Almaty, Kazakhstan

E-mail: vmsoms@rambler.ru

- Received Date: 18 July 2024
- Accepted Date: 29 July 2024
- Publication Date: 31 July 2024

Keywords

mechanics, dynamics, dissipation, energy, entropy, evolution.

Copyright

© 2024 Authors. This is an open-access article distributed under the terms of the Creative Commons Attribution 4.0 International license.

Citation: Somsikov VM. The Role of the Structure of Matter in its Dynamics and Evolution. Japan J Res. 2024;5(6):046.

numerically simulating systems of colliding disks [7,11]. As a result, it was found that interacting disk systems actually tend to equilibrium. But such models of systems are too simplified and do not allow obtaining analytical expressions necessary for understanding the processes of establishing equilibrium. Therefore, as a next step, it was decided to study systems of potentially interacting MPs as structured bodies (SB). It was clear that if we obtain the equation of motion of such systems, we can take into account and understand the role of the structure of bodies in their dynamics.

It turned out that the equation of motion SB cannot be obtained within the framework of the canonical formalisms of classical mechanics, since they are limited by the requirement of holonomy of constraints [7]. To obtain it, additional concepts were proposed, dictated by the structure of bodies. These, in particular, include the concept of dualism of the symmetry of body and space. According to this dualism, the dynamics of SB are determined not only by the symmetries of space, but also of the body itself. The symmetry of a body corresponds to internal energy, and the symmetry of space corresponds to the energy of motion. The sum of these energies represents the total energy SB. The equation of motion of SB follows from the condition of its invariance. Unlike the equation of motion of the MP, this equation takes into account that the work of external forces when moving bodies in inhomogeneous space, goes not only to their movement, but also to changing internal states. To determine the magnitude of this part of the work, the concept of D-entropy was introduced. The proposed extension of classical mechanics opened up the possibility of describing dissipative processes of evolution [5, 7]. It also affected other branches of physics, in particular, quantum mechanics [13]. Its linearity and connection with the concepts of probability are still controversial [14-16]. In general, the expansion of classical mechanics led to the need to build "Physics of Evolution", defining its main tasks as solving the problems of the emergence, development and decay of bone matter systems within the framework of the fundamental laws of physics [7].

In this work we will explain how and why taking into account the structure of matter in its dynamics and interactions opens up the possibility of describing the processes of the evolution of matter. To do this, we will discuss the principles on which the equation of motion SB is built, as it follows from the concepts of symmetry and the condition of invariance of the total energy, its properties. Let us discuss the key concepts and formulas that formed the basis for the construction of the Physics of Evolution. Let's consider how the contradictions between classical mechanics and thermodynamics are eliminated and what opportunities this opens up for the development of physics.

Energy of structured bodies

Equation of motion Newton allows you to solve a huge number of practical physics problems. The canonical equations of Lagrange and Hamilton, which underlie the foundations of classical mechanics, obtained based on the equation of motion of MPs under the conditions of the potentiality of collective forces and holonomy of connections, they also make it possible to establish the fundamental laws and dynamics of bodies. However, the limitations of classical mechanics, due to the use of a structureless model of bodies, do not allow taking into account dissipative forces, for example, friction forces. However, it is dissipative forces that determine the processes of evolution. Therefore, they had to be taken into account empirically and statistically.

Let us consider the essence of dissipative forces using a simple example of the movement of a body along an inclined rough surface. In the simplest case, this movement is described by an equation in which a friction force is added proportional to the speed of the body. It has the form [17]:

$$MV_0 = -F_0 - \alpha V_0 \quad (1)$$

where M is the body mass, V_0 is the velocity of the center of inertia (CI), F_0 is the force acting on CI, α is the friction coefficient.

Usually, the coefficient of friction is obtained from experience. It can also be found within the framework of statistical physics, based on the theory of fluctuations [17]. However, the application of the theory of fluctuations is limited to systems slightly removed from equilibrium.

The problem of a body sliding along an inclined rough surface (see Figure 1) is interesting because in it the state of the body changes when interacting with the sliding surface as a result of movement, which is a necessary element of evolution.

The total energy of a body is equal to the potential energy of gravity. In the process of moving a body, potential energy is converted into its energy of motion and into heating. In accordance with equation (1), the body will stop when, as a result of the work of friction forces, its entire energy of motion is converted into internal energy. Thus, the experiment shows that the violation of time symmetry is associated with the irreversible transformation of the energy of motion of bodies into internal energy. Such a transformation occurs under the condition that the movement of each MP of the body is subject to Newton's equation. But at the same time, the motion of the body does not obey Newton's equation, since the work of external forces changes not only the energy of motion of the body, but also the internal energy.

Equation (1) takes into account the role of the internal structure of the system in its dynamics, which manifests itself through friction forces. But to understand the friction mechanism, it is necessary to have its analytical expression. This will make it possible to understand the nature of friction and bring us closer to understanding evolution, which is impossible without dissipation. Let's do this by defining a body model as a system of potentially interacting MPs, taking into account that the total energy of the body is the sum of the energies of all its MPs.

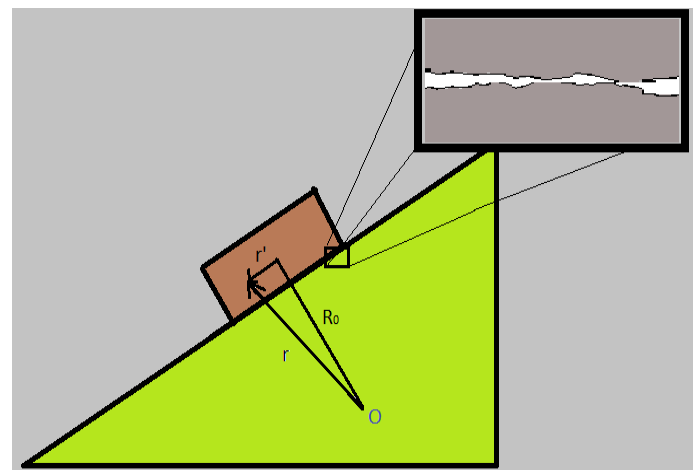


Figure 1. Slipping of the body under the influence of force gravity on an inclined surface with friction

In Figure 1 shows a body model in the form of an MP system. Vector r determines the position of one of the MP systems in the laboratory coordinate system with the origin at point O . R_0 - radius vector CI of the body, r - position of the selected MP, r' - radius vector of this MP relative to CI. The radius vector MP can be written as a sum of vectors: $r = R_0 + r'$. In the MP velocity vector in this coordinate system, which we will call *dual*, has the form: $v = V_0 + v'$.

In a dual coordinate system, the work of external forces breaks down into the work of changing the speed CI and the work of increasing the internal energy. The variables that determine the movement of CI will be called macrovariables, and the variables that determine the movement of MP relative to CI will be called microvariables. It is important that the micro and macro variables that determine the movement of the MP and the movement of the body, respectively, are independent. Microvariables are determined by the forces of interaction between the elements of the system, and macrovariables are determined by external forces independent of internal forces. Internal energy is determined by the symmetries of the body, and the energy of motion is determined by the symmetries of space. This means that the dynamics of a structured body and changes in its internal state are determined by the relationship between the symmetry of the system and the symmetry of space. Therefore, the study of the question of the role of the structure of bodies in their dynamics should be based on the concept of symmetry dualism [18,19].

It is known that Newton's equation of motion can be obtained from the condition of energy invariance. It is obvious that the equation of motion SB can be found in a similar way, but taking into account the structure, symmetry of the body and, accordingly, its internal energy. In this case, we must proceed from the fact that the dynamics of all MPs in SB obeys Newton's laws. The task is that, knowing the properties and laws of behavior of elements, obtain the properties and laws of behavior of their systems.

In accordance with the concept of symmetry, to study the relationship between the energy of body motion and internal energy, we will represent the total energy of the system in a dual coordinate system through independent groups of micro- and macrovariables [20]. The essence of the principle of symmetry dualism for a system can be explained using Figure 1. It shows that each MP of the body participates in movement relative to CI and in movement together with the whole body. That is, each MP contributes to the energy of motion and internal energy. The movement of the MP relative to CI is determined by the potential interaction forces between the MPs, which depend on the distances between them. The movement of CI is determined by the total external force applied to each MP. This force is determined by the CI coordinates in space.

We will determine the energy of the body in accordance with the molecular kinetic theory for an equilibrium system of N potentially interacting MPs. Since the system has finite dimensions, in the inhomogeneous field of external forces created in our case by a rough surface, the forces acting on its side on different elements of the system will be different. If in this case the interaction forces of the MP turn out to be commensurate with the differences in the external forces acting on them, then the external forces will lead to a change in the energy of the relative motion of the MP. This is equivalent to a change in internal energy.

The internal energy is a component of the invariant total

energy of the system. Therefore, it is obvious that the change in internal energy will occur due to energy of body movement.

Let us obtain an expression for the total energy SB in the dual coordinate system.

Under the condition of additivity of energy, the total energy SB is determined by the sum of energies MP. The energy of each MP consists of its energy of motion, the potential energy of interaction of all MPs and the potential energy of each MP in the external field of forces. Each MP, in addition to moving together with the system, is in relative motion to each other as a result of the work of their interaction forces and the difference in external forces acting on the elements of the system. These movements determine internal energy.

In the laboratory coordinate system, the total energy has the form:

$$E_N = T_N + U_N^{ins} + U_N^{env} = const. \quad (2)$$

Here $T_N = \sum_{i=1}^N mv_i^2 / 2$ is the kinetic energy of the system, equal to the sum of the kinetic energies of all MPs; $i = 1, 2, 3, \dots, N$; v_i - speed i -th MP; $U_N^{ins} = \sum_{i=1}^{N-1} \sum_{j=i+1}^N U_{ij}(r_{ij})$ - potential energy of MP interactions; $r_{ij} = r_i - r_j$ - distance between i and j MP; $U_N^{env} = \sum_{i=1}^N U_i^{env}$ - potential energy of the system in the field of external forces, determined by the sum of potential energies of each MP; m - MP mass, which we take equal to 1.

Let us write equation (2) in the dual coordinate system as the sum of the energy of motion of the system and internal energies. Provided that the external force field is uniform, micro and macro variables are separated, and the energy expression takes the form:

$$E_N = E_N^{tr} + E_N^{ins} = const \quad (3)$$

Here $E_N^{tr} = E_N^{tr} + E_N^{env}$ is the energy of motion of the system; T_N^{tr} - kinetic component of the energy of motion of the system; $E_N^{ins} = T_N^{tr} + U_N^{env}$ - internal energy, where T_N^{tr} ins is the kinetic component of internal energy.

Equation (3) - dual representation of energy body in the form of the sum of the energy of its movement and internal energy. Here it is so far written under the assumption that the potential component of the internal energy does not depend on the radius vector CI of the system. As we will see, this is true only when the scales of inhomogeneities in the external fields are much larger than the scales of the system. Otherwise, the external field of forces will contain terms determined by micro- and macrovariables. That is, there will be a "linkage" between micro and macro variables. It is these terms that determine the relationship between the energy of motion and internal energy.

According to Noether's theorem, different types of motion correspond to different types of symmetry. For example, different types of symmetry include rotation and translation of the body. For SB, specified by a set of MPs, this means that its dynamics should be determined not only by the symmetry of space, as in the case of one MP, but also by the symmetry of the body itself, composed of a set of MPs.

In accordance with the principle of symmetry dualism [7], to describe the dynamics of a body, a dual representation of energy is necessary. From the condition of additivity of mass and the fact that the momentum of the system corresponds to the movement of a body with a total mass and speed determined by the total speed of the elements of the system, it follows that the energy and movement of the system is determined by the speed CI. It looks like: $v_N = (\sum_{i=1}^N v_i) / N$ [3]. To represent the total energy in

micro and macro variables, let's use the formula that is valid for quadratic functions. This formula for kinetic energy of a system of N particles, written in a laboratory coordinate system, has the form:

$$N \sum_{i=1}^N v_i^2 = \left(\sum_{i=1}^N v_i \right)^2 + \sum_{i=1}^{N-1} \sum_{j=i+1}^N v_{ij}^2 \quad (4)$$

Where $v_i - v_j = v_{ij} = r_{ij}$. From here we have:

$$T_N = M_N V_N^2 / 2 + [m / (2N)] \sum_{i=1}^{N-1} \sum_{j=i+1}^N v_{ij}^2 \quad (5)$$

where $M_N = \sum_{i=1}^N m_i = mN$

That is, the quadratic form of kinetic energy in the laboratory coordinate system is also a quadratic form in the dual coordinate system. The first term on the right-hand side of equation (5) is the kinetic energy of the system's motion. The second term corresponds to the kinetic component of the internal energy of the system. Thus, the total kinetic energy of the system naturally breaks down into the kinetic energy of motion CI and the internal kinetic energy of relative motions MP. The forms of the kinetic energy of motion of the system and its kinetic component of internal energy in the dual coordinate system turned out to be invariant to the form of the kinetic energy of the system in the laboratory coordinate system. The fact that the kinetic energy of the system in micro- and macrovariables decomposes into the sum of independent quadratic functions of velocities follows from the independence of the micro- and macrovariables.

Converting energy T_N by replacement: $v_i = V_N + \tilde{v}_i$ where \tilde{v} where are the velocities of particles relative to CI. Since $\sum_{i=1}^N \tilde{v}_i / 2 = 0$, we get: $T_N = M_N V_N^2 / 2 + \sum_{i=1}^N m \tilde{v}_i^2 / 2$. Here the second term expresses the kinetic energy of motion of all MPs relative to CI. Since this expression for the total kinetic energy must coincide with expression (5), we obtain: $\sum_{i=1}^N m \tilde{v}_i^2 / 2 = [m / (2N)] \sum_{i=1}^{N-1} \sum_{j=i+1}^N v_{ij}^2$ (a). That is, the second term in (a) is equal to the kinetic energy of motion of the MP relative to CI: $T_N^{ms} = [m / (2N)] \sum_{i=1}^{N-1} \sum_{j=i+1}^N v_{ij}^2 = \sum_{i=1}^N m \tilde{v}_i^2 / 2$. From here we have:

$$T_N = M_N V_N^2 / 2 + \sum_{i=1}^N m \tilde{v}_i^2 / 2 \quad (6)$$

Thus, the total kinetic energy of the system T_N includes the energy of its motion - T_N^r and the internal component of the kinetic energy, determined by the energies of motion of all MPs relative to CI - T_N^{ms} , i.e. $T_N = T_N^r + T_N^{ms}$, Where $T_N^r = M_N V_N^2 / 2$, $T_N^{ms} = \sum_{i=1}^N m \tilde{v}_i^2 / 2$. Consequently, the internal energy of the system can be written both in the relative coordinates and velocities of the MP, and in the coordinates and velocities of the MP relative to CI.

Let us write the total energy SB in the dual coordinate system:

$$E_N = M_N V_N^2 / 2 + \sum_{i=1}^N m \tilde{v}_i^2 / 2 + \sum_{i=2}^{N-1} \sum_{j=i+1}^N U_{ij}(\tilde{r}_{ij}) + \sum_{i=1}^N U_i^{imv}(R_i(R_0, \tilde{r}_i)) = const, \quad (7)$$

In equation (7) in the last term the radius is a vector R_i each MP is expressed through micro and macro variables. In the general case, micro and macro variables in external forces are not separated. This means that the external field of forces changes both the energy of motion of the entire system and its internal energy. The nature of such a change will be clear from the equation of motion SB.

Thus, we were able to obtain the expression for the total energy SB, written in a dual coordinate system. The law of conservation of the total energy of a system, in accordance with the principle of symmetry dualism, should be defined as

follows: the energy of the system, equal to the sum of the energy of motion of the system and its internal energy, changes along the trajectory of its CI so that the total energy is conserved. If the equation of motion of the MP system is obtained within the framework of the formalisms of classical mechanics and by adding the equations of motion of each MP, such a relationship will disappear [2]. This is due to the fact that the sum of all internal forces, as well as the sum of all MP impulses, is equal to zero. As a result, only the contribution of the external force remains to change the energy of motion of the system, equal to the sum of the external forces. This explains the reversibility of the equation of motion of the system, which follows from classical mechanics.

Energy of an open nonequilibrium dynamic system

Above, we obtained an equation for the energy SB in an external inhomogeneous force field. But in nature there are neither absolutely equilibrium systems nor absolutely structureless elements. All systems are ONDS. In the approximation of local thermodynamic equilibrium, ONDS are a set of SBs moving relative to each other in the inhomogeneous self-consistent field of forces they create and in the field of external forces. We will call the representation of the body in the form of SB the first level of description of the ONDS, and its representation in the form of a set of SBs the second level of description of the ONDS. This level is used in statistical physics. There it is studied based on the distribution function [17,21].

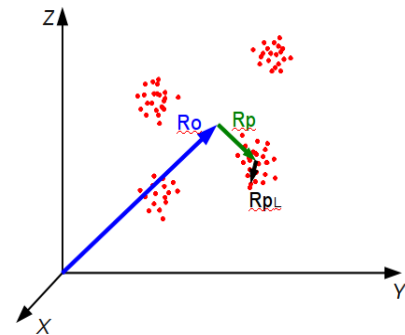


Figure 2. Second level of description of ONDS.

For the second level of description of the ONDS, the movement of SB will be determined by the inhomogeneous field of forces created by SB and the external force field. Therefore, to describe their dynamics it is necessary equation of motion for a set of interacting SBs in a non-uniform force field. It will allow us to take into account the division of the work of external forces into the movement of SB and their change their internal energy

In Figure 2 shows a diagram of the second level of description of the ONDS for four SBs. Here R_0 - CI for ONDS, Radius - vector R_p denotes CI for subsystems consisting of P-micro material points. Red dots are micromaterial points that make up subsystems. R_{pl} - position of micromaterial points MP relative to the CI subsystem.

The energy of the second level of description of the ONDS can be determined if we use the expression for the energy SB, but at the same time each of its MPs is considered a subsystem consisting of potentially interacting micro-MPs with the same masses - μ . To do this, the potential energy of interaction of subsystems should be written taking into account the self-consistency of the movement of all interacting elements.

From the condition of infinitely divisible matter [7], for the

following levels of description of the ONDS, each micro-MP can be represented as a system of smaller particles and so on. There may be an infinite number of such hierarchical levels of matter.

Let the ONDS consist of N subsystems, with each subsystem containing the same number of micro -MPs μ equal P . Let the mass of micro -MP “ μ ” equal to one. Then the mass of the MP is equal to $m = \mu P$. The mass of the entire ONDS will be equal to $M_N = \mu NP$, and NP is the number of all μ included in the ONDS. Let's write down the energy equation for the second level of description of matter. It looks like:

$$E_{ONDS} = M_N V_N^2 / 2 + \sum_{i=1}^N m \tilde{v}_i^2 / 2 + \sum_{i=1}^N \sum_{k=1}^P \mu (\tilde{v}_k^i)^2 / 2 + \sum_{i=1}^N \sum_{k=1}^{P-1} \sum_{s=k+1}^P U_{ks}^i(\tilde{r}_{ks}^i) + \sum_{i=1}^{N-1} \sum_{l=i+1}^N \sum_{k=1}^P \sum_{s=1}^P U_{ks}^{il}(\tilde{r}_{ks}^{il}) + \sum_{i=1}^N U_i^{jmv}(R_i) = const \quad (8)$$

Here N is the number of subsystems, with $i = 1, 2, 3 \dots N$, P is the number of micro -MPs in the subsystems, with $k = 1, 2, 3 \dots P$; V_N - speed CI ONDS; $m = \mu P$; \tilde{v}_i - speed CI of the i - th subsystem relative to CI ONDS, and $\tilde{v}_i = (\sum_{k=1}^P v_k^i) / N$; $U^i(\tilde{r}_{ks}^i)$ potential energy of the i - th subsystem, due to the interactions of its micro - MP; $\tilde{r}_k^i, \tilde{v}_k^i$ - coordinates and speeds of micro -MP relative to the CI i - th subsystem, and $\tilde{r}_{ks}^i = \tilde{r}_k^i - \tilde{r}_s^i$ the distances between them, indices k, s vary from 1 to P and refer to micro -MP for the i - th subsystem; all subsystems contain the same number of micro -MPs; $\tilde{r}_{ks}^{il} = \tilde{r}_k^i - \tilde{r}_s^l$ - distances between micro-MPs k and s subsystems i and l . This distance should be represented as the sum of the radius vector, which determines the distance between the CI of the subsystems, and the radius-vectors of the micro-MP positions relative to the CI of the corresponding subsystem.

Expression (8) determines the total energy of the ONDS in a non-uniform field of external forces, which consists of micro-MP subsystems.

The first term in (8) is the kinetic energy of the movement of the ONDS in the field of external forces.

The second term is the kinetic energy of movement of the subsystems relative to CI ONDS.

The third term is the internal kinetic energy of the subsystem relative to their CI.

The fourth term determines the total internal potential energy of the subsystems.

The fifth term determines the potential energy of the second level of description of the ONDS, determined by the interactions of micro-MP of all subsystems.

The sixth term determines the potential energy of subsystems in the field of forces external to the ONDS. In it, the radius-vector R_i determines the position of the i - th subsystem in the laboratory coordinate system. This radius-vector is the sum of the radius-vector CI of the ONDS and the radius-vector CI of the i - th subsystem.

It is obvious that, as in the case of SB, the equation of the second level of description of the ONDS follows from the energy equation of the ONDS (8). Due to the new types of energies in the ONDS, an additional hierarchical link of micro- and macrovariables will appear in its equation of motion. Corresponding to this hierarchy is the hierarchy of forces.

In a similar way, it is possible to obtain an equation for energy up to any level of ONDC, but this is not necessary. Indeed, in the general case, the evolution of the ONDS will be determined by the linking of variables of all levels of the ONDS. But nature

dictates the approximation according to which the linking of variables is significant only for adjacent hierarchical levels. That is, the work of external forces decreases significantly for each subsequent level. This is due to the hierarchy of scales of the levels of ONDS. As a rule, the characteristic scales of inhomogeneities in the interaction forces of subsystems of a given level of description are commensurate with the scales of these subsystems, but are significantly larger than the scales of the next level of description. That is, with a sufficient degree of approximation, we can assume that the work of external forces in relation to a given level of description of the ONDS changes the energy of motion of a given level of description of the ONDS and the energy of the relative motion of its subsystems, but does not change the internal energy of the subsystems. Moreover, in nature, the hierarchy of forces is built in accordance with molecular, atomic, nuclear and other forces. Due to the great difference between these forces, there is a stable hierarchy of matter structures: molecules, atoms, nucleons, etc. An additional argument in favor of these conclusions is that entropy has a second degree of smallness [17]. This argument will find its confirmation in the equation of motion SB obtained below. Thus, to describe evolution, it is enough to find the equation of motion SB.

Equation of motion SB

In addition to using the principles and fundamental laws of classical mechanics, the derivation of the SB equation of motion relied on additional principles dictated by the structure of matter and symmetries. These include the following principles [22]:

the laws for the body and its environment, as well as their physical characteristics, are equivalent; the dynamics of systems is determined by the principle of symmetry dualism;

evolution is determined by the nonlinear relationship between the symmetries of the medium and SB.

Without these principles it is impossible to obtain the equation of motion SB. This equation is derived from the condition of invariance of the total energy, which takes into account that the work of external forces occurs as a change energy of motion SB, determined by the symmetries of space, and on the change in its internal energy, determined by the symmetries SB. That is, the total energy, equal to the sum of the energy of motion and internal energy SB, is not only an invariant of motion, but also an invariant of evolution SB. Neither the energy of motion MP, nor the internal energy separately are no longer invariants and do not determine evolution. Let us show how to obtain the equation of motion SB from the energy equation (7).

According to Noether's theorem, the symmetries of space and time determine the total energy of MP. For the equation of motion of a MP, the total energy coincides with the energy of its motion, since the MP has no internal energy. The equation of motion of the MP follows from the invariance of the energy of motion by differentiating it with respect to time. The equation of motion SB, as well as the equation of motion MP, should follow from the invariance, but of the total energy, equal to the sum of the energies of the MPs included in it. It breaks down into energy of motion and internal energy SB. The equation of motion SB follows from the total energy (7), presented through micro- and macrovariables in the form of the sum of the energy of motion and internal energy by differentiating it with respect to time and requiring its invariance. It has the form [7]:

$$M_N V_N = F_N^0 - \mu^d V_N, \quad (9)$$

Where $F_N^0 = \sum_{i=1}^N F_i^0$; F_i^0 - external force acting on i - th MP; $\mu^d = E_N^{\text{int}} / (V_N^{\text{max}})^2$; F_{ij} - interaction force i and j MP; $F_{ij}^0 = F_i^0 - F_j^0$;
 $E_N^{\text{int}} = \sum_{i=1}^{N-1} \sum_{j=i+1}^N v_{ij} (mv_{ij} + F_{ij}^0 + NF_{ij}^0)$; $V_N^{\text{max}} = -E_N^{\text{int}} / F_N^0$

Equation (9) takes into account the relationship between the movement of elements SB relative to CI and the movement CI with SB. As a result, it takes into account that when a body moves in a non-uniform field of external forces, their work goes both to change the body's motion energy and to change its internal energy.

On the right-hand side of equation (9), the first term determines the external potential forces that change the speed CI. Its appearance is due to the symmetries of space. The second term is determined by both the symmetries of space and the body, depends on micro and macro variables and is non-zero when there is a gradient of external forces, that is, when: $F_{ij}^0 = F_i^0 - F_j^0 \neq 0$, given that $\sum_{i,j} F_{ij}^0 = 0$. It is called evolutionary nonlinearity, since it determines evolution, determining changes in the internal state of the body depending on the dynamics of the body. The nature of evolutionary nonlinearity is connected with the fact that due to the heterogeneity of the external field of forces and the structural nature of the system, an interconnection of micro- and macrovariables arises. That is, evolutionary nonlinearity determines the relationship between the energy of motion SB and its internal energy and how the state of SB changes in accordance with the symmetries of the body and the environment. In the linear approximation, evolutionary nonlinearity disappears, and equation (9) becomes Newton's equation of motion. That is, equation (9) satisfies the correspondence principle.

According to equation (9), symmetry is broken not according to the scenario that is interpreted, for example, in particle physics [19]. It simply does not persist due to the interconnection of micro- and macro-descriptions at special points. That is, here we have exactly that case of violation of the symmetry of time, which is determined by the transition of "Order" to "Chaos". The energies of movement SB are put in correspondence with the measure of "Order", and the internal energy is put in correspondence with the measure of "Chaos" [23]. "Order" is determined by the sum of MP speeds, and "Chaos" is determined by the relative MP speeds at which the sums of MP impulses and their interaction forces are equal to zero. This can be considered the essence of the concepts of "Chaos" and "Order". That is, through "Chaos" nature realizes the possibility of the existence of bodies at rest, when to the movement of their elements. If "Order" is characterized by the concept of energy of motion, then "Chaos" is characterized by the concept of entropy, which is determines the change of internal energy. The qualitative difference between "Order" and "Chaos" corresponds to the qualitative difference between the concepts of energy and entropy. A system is in a state of rest and equilibrium if a subsystem identified in it in any way is motionless relative to all other subsystems of the system. To determine the measure of transformation of the energy of "Order" into the energy of "Chaos," the concept of D-entropy was introduced. It is determined by the ratio of the increment of internal energy to its value.

Let us consider when and why the dynamics of a system in a non-uniform field of external forces becomes irreversible. Here, it would seem that the easiest way to explain irreversibility is that the work of external forces is divided into changes in

internal energy and energy of motion. Since matter is divisible indefinitely, such division of work will occur across all its hierarchical levels. But, as it turned out, for sufficiently large systems the explanation of irreversibility follows from equation (9).

In accordance with equation (9), the increase in internal energy is due to evolutionary nonlinearity. This means that it has a second order of smallness. A reverse flow of internal energy occurs when the SB equilibrium is disturbed due to strong inhomogeneities of external forces. But it is also determined by evolutionary nonlinearity and by equation (9). Therefore, it does not exceed the fourth order of smallness. Hence, the energy balance of SB is determined by the expression [7,22]:

$$\Delta E_{\text{dec}}^{\text{ir}} \approx \alpha \chi^2 - \beta \chi^4 \quad (10)$$

Here α , β constants defined by equation (9). That is, the increase in internal energy for a sufficiently large SB is always positive, which proves irreversibility for small ones χ . It turned out that the quantity obeys the law $1/\sqrt{N}$ [17, 25].

The presence of dissipation is a necessary condition for the formation of attractors, and therefore for the formation of attractors systems. From the condition that dissipation is possible only for SB, it follows that if the world arose in an evolutionary way, then matter should be infinitely divisible and represent an infinite hierarchy of interconnected SB, when any no matter how small selected part of SB is also SB [25].

Since the connection between the levels of description of the second-order ONDS is small, equation (9) can be used with sufficient accuracy to describe the evolution of the ONDS.

We emphasize that the principle of dualism of work for SB, which automatically follows from the principle of symmetry dualism, like Galileo's principle, refers to the fundamental principle that determines the behavior of real bodies. Indeed, the work of external forces on SB consists of the work of moving all MPs. And this work consists of both the work to change the speed module of the MP and the work to change its direction. Moreover, the change in the direction of the MP velocity contributes to the chaotic component of the SB energy. That is, the work of external forces changes the contribution of each MP to the energy of "Order" and to the energy of "Chaos". This leads to a violation of the invariance of motion energy SB, which determines the reversibility of time in classical mechanics. Thus, violation of the invariance of the energy of motion is possible only when the energy of motion can be transformed into other forms of energy; in this case, it turns into the internal energy of the chaotic motion of the MP. That is, the structural nature of matter not only expands classical mechanics, but qualitatively changes its foundations, introducing evolution into it.

Extended classical mechanics is based on the invariance of total energy and its representation as the sum of the energy of motion and internal energy. In this case, the internal energy increases due to the energy of motion. This suggests the idea of introducing the concept of complete phase space for the study of evolutionary processes. It represents the sum of phase subspaces for the energy of motion and internal energy. Such a dual phase space is built on the space of micro- and macrovariable coordinates and momenta of the SB and its elements [7]. According to equation (9), the evolution of bodies is determined by how the internal energy and the energy of motion are related through evolutionary nonlinearity. It is characterized by the ratio of the magnitude of changes in internal energy to its total value. This quantity is called D-entropy [25]:

$$\Delta S_N^d = \Delta E_N^{\text{int}} / E_N^{\text{int}} \quad (11)$$

The fundamental difference between D-entropy and existing entropies is that it is deterministic and determines the change in the internal states of moving systems in an external field of forces. For small SB, for example, for an oscillator, D-entropy can be either positive or negative [26]. But already at $N_1 > 100$ D-entropy only increases. That is: $\Delta S_{100}^d > 0$. When $N_2 > 10^3$ the increment of D-entropy reaches asymptotics. That is, $N_2 \sim 10^3$ determines the range of applicability of the thermodynamic description of the system. A similar conclusion follows from statistical laws [30]. In general, these critical numbers depend on the parameters of the task.

As a test of the equation of motion (9), a particular three-body problem was solved. It considers the motion of an oscillator in a non-uniform force field [27]. It turned out that taking into account the duality of the work of external forces leads to the possibility of an oscillator passing through a potential barrier even when the height of the barrier is greater than its energy of motion, but less than the total energy. That is, such passage is carried out due to internal energy. This effect was confirmed when studying the behavior of D-entropy depending on the number of particles in the system [27].

If we have an ONDS of the second level of description, then its study can be carried out on the basis of the extended Liouville equation [7,21,28]:

$$df_i / dt = \partial f_i / \partial t + \sum_{i=1}^N \{V_i(\partial f_i / \partial R_i) + P_i(\partial f_i / \partial P_i)\} = -f_i \sigma \quad (12)$$

Here $i = 1, 2, 3 \dots N$ - SB number; F_i - forces acting on i -th SB; R_i, P_i - coordinates and momentum SB; $f_i = f_i(R, P, t)$ distribution functions for subsystems; $\sigma = \sum_{i=1}^N \partial F_i / \partial P_i$

Equation (12) was obtained in the same way as the canonical Liouville equation [17], but to obtain it, instead of Newton's equation, equation (9) was used.

The scope of use of equation (9) is wider than the scope of use of the canonical Liouville equation. It does not require the ensemble to be close to the equilibrium state. Moreover, it describes the transition of the energy of motion of SBs into their internal energy, provided that the total energy of the ensemble and its total phase volume are conserved. It takes into account the work of dissipation forces and is applicable to describe evolution.

From equations (9-12) it follows that the ONDS tends to an equilibrium state due to the transition of the energy of relative movements of SB into their internal energy.

For the entropy at rest SB close to equilibrium, the Boltzmann formula is applicable: $S^B = -\int (f_i \ln f_i) dp dq$

From (12) it follows that if $0 \sigma = 0$, then $dS^B / dt = 0$. Therefore, S^B it has a maximum for the equilibrium ensemble. A similar result was obtained in statistical physics from the requirement that the entropy of equilibrium systems be extremal [17]. That is, according to equation (12) the closed ONDS comes to an equilibrium state because the transfer of energy and relative motions of SB into their internal energy. Consequently, the existence of stationary ONDS is possible only in the presence of external force fields that provide a stationary influx of energy, compensating for the dissipation of energies of relative motions SB. Here it should be taken into account that, within the framework of the laws of modern physics, the existence of stationary ONDS can only be explained by Planck radiation, which ensures the balance of inflow and outflow of energy into

the ONDS.

Thus, thanks to the structure of models of bodies, an extension of classical mechanics and the physics of evolution arises. The task of the physics of evolution is to describe the processes of emergence, development and decay of systems within the framework of the fundamental laws of physics. The possibility of describing evolutionary processes is due to taking into account changes in the internal states of bodies when they move in inhomogeneous fields of external forces. New concepts in the Physics of Evolution are D-entropy. It is also based on the extended Liouville equation, the concept of complete dual phase space, and the modified Lagrange, Hamilton, and Hamilton-Jacobi equations [7]. In these equations, thanks to taking into account the structural nature of bodies, an additional term appeared, depending on micro and macro variables. Today, the absence of this term in canonical equations is compensated in practice by empirical additions, which are determined from experiment. This, for example, is done in quantum mechanics when solving the problem of hidden variables [14].

Let us emphasize that according to equation (9), the evolution mechanism is deterministic [7,28]. This is consistent with the opinion of Planck, who was confident that there is a deterministic interpretation of the second law of thermodynamics, and the statistical interpretation used is a consequence of the deterministic one [29]. That is, the scope of use of the statistical form of the second law of thermodynamics, as well as statistical laws, is determined by equation (9).

Structurality of matter and some problems of physics

Let us discuss how taking into account the structure of bodies in their dynamics can contribute to the development of physics. First of all, equation (9) plays a key role in this. It follows from it that taking into account the structure of bodies in their dynamics removes the contradiction between mechanics and thermodynamics. The concept of D-entropy plays an important role here. It connects the movement of bodies and changes in their internal energy. Its use does not require the fulfillment of the condition that the systems are close to equilibrium. D-entropy and energy of motion characterize evolution and determine the range of applicability of thermodynamic and statistical descriptions. The extended Liouville equation (12) can play an important role in statistical physics and kinetics, since it is applicable to the analysis of nonequilibrium systems. The canonical Liouville equation was obtained for the stationary state of the system, when the probability distribution density in the phase space or the statistical distribution function does not change with time. It describes the behavior of systems near equilibrium, when dissipative processes are negligible. Therefore, it follows from it that the distribution function is conserved along the phase trajectories of the system. This also yields a microcanonical distribution applicable for systems at rest [17]. But the canonical Liouville equation cannot be used to study the evolution of ONDS [17].

Questions arise related to Fermi-Dirac and Bose-Einstein statistics [17, 21]. They were obtained for systems close to equilibrium. How fair they are for ONDS is an open question.

Especially the physics of evolution can be useful for understanding and developing quantum mechanics. It was built for elementary particles, based on experimental facts. According to these facts, the dynamics of microparticles satisfies the principle of wave-particle duality. Moreover, the energy and their other dynamic parameters are quantized, that is, they

are discrete in nature. Naturally, the theory of the microworld was built based on the requirement that it satisfy these facts. According to them, the fundamentals of quantum mechanics use the wave function $\psi = \psi(x, y, z, t)$. The square of the modulus $|\psi|^2$ determines the probability of a particle being in a given region of configuration space $dV = dx dy dz$. The wave function of a free particle has the form of a plane monochromatic wave de Broglie [13-16]: $\psi_i(r, t) = A e^{i(kr - \omega t)/\hbar} = A e^{i(p \cdot r - Et)/\hbar}$, where $k = p/\hbar$ satisfies the Schrödinger equation [13]:

$$i\hbar \frac{\partial}{\partial t} \psi = \left\{ -\frac{\hbar^2}{2m} \nabla^2 + U(r, t) \right\} \psi \quad (13)$$

Equation (13) was obtained based on the principle of least action for a microparticle. In quantum mechanics, any dynamic quantity, for example, energy and momentum, is determined using the corresponding operator: $E \rightarrow i\hbar \partial / \partial t$, $p \rightarrow -i\hbar \nabla$ acting on the wave function ψ . It is determined by averaging: $\bar{f} = \int \psi^* f \psi dq$. In this case, ψ satisfies the Heisenberg uncertainty relation: $p q - q p = -i\hbar$. That is, Poisson brackets for coordinates q and impulses p for a microparticle are different from zero and coincide in magnitude with \hbar . This implies the impossibility of simultaneously accurately determining the coordinates and momenta of a microparticle. Quantum mechanics satisfies experimental facts well. However, its interpretation is still controversial. Thus, Einstein and Schrödinger did not accept Bohr's assertion that the uncertainty relation is a given of the microworld. After all, it violates the principle of causality, which lies in the foundations of the cognizability of the world. On this occasion, Einstein said the catchphrase that "God does not play dice" [32].

In connection with the established fact of the infinite divisibility of matter, the obvious question arises: how will this affect quantum mechanics? After all, the structure of particles requires taking into account their internal energy during interactions and decays. Structurality will lead to the fact that the motion of, for example, an electron will be a combination of two processes: the motion of its CI in space and some kind of change in its internal state. In addition, due to the structure, rotational energy may appear. That is, most likely an electron or photon is an oscillator, and their trajectory is characterized by both CI motion and oscillation and rotation. From the principle of dualism of the work of external forces on particles, it follows that during their interactions, the work of external forces will change both the energy of motion and the internal energy. This means that neglecting the structure of particles in calculations leads to errors determined by the role of structure in the change in internal energy. The need to take this circumstance into account requires transforming the Schrödinger equation so that it takes into account the dualism of [31]. In the simplest case, such a transformation is reduced to representing the Hamiltonian as a sum of two parts corresponding to the internal energy and the energy of its motion in external fields. We emphasize that these ideas correspond to the assumptions of Einstein and Schrödinger that quantum mechanics is not complete [32-35]. They also fit into the de Broglie-Bohm theory [36].

As in quantum mechanics, in electrodynamics and the theory of gravity, significant difficulties arise due to the fact that the Coulomb or gravitational potentials for a body model in the form of MP have a feature of the type $1/r$. Obviously, this feature is due to the use of a point model of a body, in particular, an electron. But such a model is approximate, since all material objects have a structure at all hierarchical levels, which means they have finite

dimensions. It is obvious that at distances commensurate with this scale, the $1/r$ approximation is wrong. Another difficulty is related to the reversibility of Maxwell's equations, which Planck encountered [34]. An example of inhomogeneous external forces are gravitational fields in the Universe. According to equation (9), their inhomogeneities lead to the conversion of part of the energy of motion of objects in the Universe into their internal energy, which determines their evolution. This effect was called "gravitational friction". "Electromagnetic friction" is defined in a similar way [21,24]. According to equation (9), the energy of an electromagnetic wave, passing through the plasma, will transform into the internal energy of the electron gas due to an increase in the chaos energy of electrons field. This will lead to heating of the plasma and attenuation of the wave in proportion to the gradients of forces acting on the electrons from the electromagnetic wave [24].

The main part of the problems that can be resolved by taking into account the structure in the dynamics and evolution of systems is related to the explanation of the mechanism of dissipation of the energy of motion. These problems led to difficulties in studying the processes of attractor emergence. To study them, it was necessary to use either numerical methods or empirical equations. This was especially acute when constructing a model of the Universe, for which it is important to understand the mechanisms of formation of star systems and galaxies, although numerical calculations largely eliminate these problems.

A major problem faced in studying the evolution of the Universe is related to hidden mass. Its essence is that the observed accelerations of objects in the Universe are lower than they should be according to Newton's equation of motion. One of the well-known solutions to this problem [37] proposes to multiply the observed mass of an object by a factor greater than one, which is precisely equivalent to the hidden mass. But if we use the equation (9) to estimate this effect, then it can be explained by the fact that the work of external forces also goes to change the internal energy SB. This is possible provided that external forces have a sufficiently large gradient, which determines the effectiveness of increasing the internal energy of the system due to the work of external forces.

It is also possible to explain the errors in energy estimates that follow from Newton's equation of motion. They can be large for non-equilibrium objects, when the energy of its motion can be drawn from internal energy. As confirmation of this possibility, we can give an example of an oscillator passing through a potential barrier. In certain cases, it passes through such a barrier due to the internal energy of the oscillator, even when its energy of motion is less than the height of the barrier, but provided that the total energy of the oscillator is greater than the height of the barrier [27].

A combined micro- and macro-description of the dynamics of SB makes it possible to understand the nature of the peculiarities of the dynamics of bodies at bifurcation points and offers an explanation of the mechanism of the butterfly effect. To describe these points, it is convenient to use the dual mapping of phase space, which allows us to take into account the role of microprocesses in the dynamics of bodies [7].

Here are given only those examples that were found in the first steps of taking into account the structure of bodies in their evolution. But already this short list, as a rule, of nonlinear problems, speaks of the prospects for the development of the theory of evolution, which arose due to taking into account the

structure of bodies in their dynamics.

Conclusions

Taking into account the role of the structure of bodies in their dynamics leads to the possibility of describing evolutionary processes within the framework of the laws of physics. This served as the basis for the construction of “Physics of Evolution”, defining its main task as solving the problems of the emergence, development and decay of bone matter systems within the framework of the fundamental laws of physics. The foundations of the physics of evolution are the SB equation of motion, extended formalisms of classical mechanics, the extended Liouville equation and D-entropy. The physics of evolution removes the contradictions between mechanics and thermodynamics. It makes it possible to solve a number of well-known problems in physics.

The description of evolution is based on the concepts of energy of motion of the system and D-entropy, which determines the efficiency of changes in the internal energy of the system. Moreover, the energy of motion is a measure of “Order”, and D-entropy is a measure of “Chaos”, corresponding to the internal structure of the body. That is, the energy of motion and D-entropy are the main parameters characterizing evolution.

From the condition of infinite divisibility of matter, it follows that it represents an infinite hierarchy of ONDS. Each hierarchical level of ONDS is characterized by D - the entropy of the SBs included in it and the energies of their relative motion. This property of matter is universal and does not depend on the characteristic scales of the system. A description of the entire hierarchical structure of the ONDS is possible in the approximation of the second level of the ONDS, based on the equation of motion SB.

The most important result of the work is that taking into account the role of the structure of bodies in their dynamics opens up a fundamental opportunity to move from studying a stationary picture of the world to constructing an evolutionary picture of the world within the framework of the fundamental laws of physics.

Funding

The work was carried out with the financial support of the BR18574080 program: “Study of fundamental and applied problems of plasma physics and plasma-like media for the purpose of solving problems of modern energy and obtaining new functional materials.”

References

- Lanczos C. The varying principles of mechanics. M. Mir. 1962.
- Goldstein G. Classical Mechanics. M. Science. 1975. 416 p.
- Landau L.D., Lifshits E.M. Mechanics. M. Science. 1973. 215 p.
- Zaslavsky G.M. Stochasticity of dynamic systems. M. The science. 1984. 273 p.
- Prigogine I. Understanding the complex. M. World. 1990. 342 s.
- Weinberg S. Dreams of a Final Theory. Vintage. New York. 1992.
- Somsikov VM. Fundamentals of Physics of Evolution. Almaty. Kaz. Nu Al-Farabi. 2021.
- Anderson PW. More Is Different. Science. New Series. 177. 1972. p. 393.
- Jou D, Casas-Vázquez J, Lebon G. Extended irreversible thermodynamics revisited (1988–98). Rep. Prog. Phys. 1999;62:1035–1142.
- Lucia U. Probability, ergodicity, irreversibility and dynamical systems. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences. 2008; 464(2093): 1089–1104.
- Sinai Ya. G. Dynamical systems with elastic reflection ergo dynamic properties of scattering billiards. Uspehi Mat. Nauk. 1970;25:141.
- Loskutov YA. Charm chaos. UFN. T.150. 2010;12:1305.
- Schrödinger A. An undulatory theory of the mechanics of atoms and molecules. Physical Review. 1926;28(6):1049-1070.
- Taschetto D, Correa da Silva R. The Dual Dynamical Foundation of Orthodox Quantum Mechanics. arXiv:2402.15031v1 [physics. hist-ph]. 23 Feb 2024.
- Hiley B, Dennis G. de Broglie, General Covariance and a Geometric Background to Quantum Mechanics. Symmetry. 2024. 16. 67.
- Greenstein J, Zajonc A. Quantum call. Modern research foundations of quantum mechanics. Dolgoprudny. Intelligence. 2012. 432.
- Landau LD, Lifshits EM. Statistical physics. T.5. M. The science. 1976. 583 pp.
- Mac Voy K. Symmetry groups in physics. UFN. 1967; 91(1): 121-150.
- Shirkov DV. 60 years of broken symmetries in physics (From Bogolyubov’s theory of superfluidity to the Standard Model). UFN. T. 2009;179(6):581-589.
- Somsikov VM. Limitation of classical mechanics and ways it's expansion. XXII International Baldin Seminar on High Energy Physics Problems. 15-20 September. PoS. 2014.
- Rumer Yu B, Ryvkin M Sh. Thermodynamics. Stat. Physics and Kinematics. M. Science. 1977. 532p.
- Somsikov VM. Physics of evolution and structure of matter. AIP Conference Proceedings 2731, 020004 (2023) DOI: <https://doi.org/10.1063/5.0133080>. Published: May 2023. P.2.
- Somsikov VM. “Order” and “Chaos” in the Evolution of Matter. 15th Chaotic Modeling and Simulation International Conference. Springer. DOI: 10.1007/978-3-031-27082-6.
- Somsikov VM. On collisionless damping of electromagnetic waves in a cloud of plasma electrons. 20th Intern. Workshop Complex Systems of Charged Particles and Their Interactions with Electromagnetic Radiation. Prokhorov General Physics Institute of the Russian Academy of Sciences. Moscow, Russia. April 8-12, 2024.
- Somsikov VM. D-Entropy in Classical Mechanics. 14th Chaotic Modeling and Simulation International Conference. Springer Proceedings in Complexity. Springer, Cham. 2021. https://doi.org/10.1007/978-3-030-96964-6_33.
- Somsikov VM., Andreev A.B. On the criteria for the transition to a thermodynamic description of the dynamics of systems. News University. Series Physics. 7. 2015.
- Somsikov VM, Denisnya VI. Peculiarities of passage of an oscillator through a potential barrier. Russian Physics Journal. 2013;56(4):463-472.
- Somsikov VM. The equilibration of an hard-disks system. IJBC. 2004; 4027-4033.
- Klein MJ. Max Planck and the beginning of quantum theory. UFN. August. 1967. 92(4):679-700.
- Aringazin AK, Mazhitov MI. Quasicanonical Gibbs distribution and Tsallis non-extensive statistics. Phys. A. 2003;325:409-425.
- Somsikov V.M. Extension of the Schrödinger equation for quantum systems. XXIII Intern. Baldin Seminar on High Energy Physics Problems. PoS. 19-25 Sept. JINR, Dubna. 2016.
- Einstein A. A note on quantum theory. Performance V discussions on 5th Solveevsky Congress. Collection scientific works. V. 4. tt. T. III. M. Science. 1966. 528–530.

33. Belinsky AIN. About David Bohm's "pilot wave" concept. UFN. 189. 2019. 1352–1363.
34. Planck M. Zur Geschichte der Auffindung des physikalischen Wirkungsquantums [The history of the discovery of the physical quantum of action]. Die Naturwissenschaften. 1943. 31 (14). 153–159.
35. Peliti L, Rechtman R. Einstein's Approach to Statistical Mechanics: The 1902–04 Papers. arXiv:1606.04890v1 [physics.hist-ph]. 15 Jun 2016.
36. Bohm D. A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. I Phys. Rev. 1952;85:166.
37. Milgrom M. A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis. Astrophysical Journal. 1983;270:365-370.