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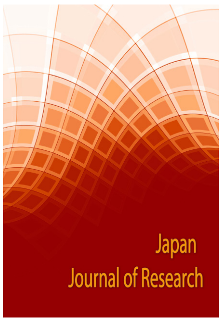

Assessment of the influence of the mean shear stress on multiaxial high-cycle fatigue of metallic materials

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Introduction

A study has been carried out to assess the influence of a superimposed mean shear stress on the capability of some multiaxial high cycle fatigue criteria. Five critical plane-based criteria, namely Matake (M), Susmel & Lazzarin (S&L), Findley (F), Carpinteri & Spagnoli (C&S) and Liu & Mahadevan (L&M) [1-5] were investigated deriving from their own equations the dependence of the fatigue resistance limit in shear stress loading as function of the mean shear stress. Seven different loading conditions reported in previous studies as critical relative to 42CrMo4 and 34Cr4 steel alloys have been considered [6]. Such loading conditions, which are expected to lead the materials to the threshold of failure in the order of one million cycles, were applied to the previously mentioned criteria (M, S&L, F, C&S and L&M), as well of being applied to a mesoscopic scale-based criterion proposed by Papadopoulos (P) [6]. Since Papadopoulos' criterion, in agreement with well-established experimental observations [7-9], is independent of mean shear stress, it seems appropriate to conclude that the inclusion of the mean shear stress in the criteria should exert a negative influence on the fatigue behaviour predictive capability. While Matake, S&L and L&M do not present a dependence of the fatigue resistance on the mean shear stress, Findley and C&S had such a dependence derived and evidenced as curve plots. Defining an error index, it is possible to use the criteria's outputs to confirm whether the models independent of mean shear stress present a better fatigue predictive capability.

Calculation procedures

Several reviews of some commonly used multiaxial high cycle fatigue damage criteria, including stress-based models, can be found in the literature [2,6,10-13]. The selected critical plane-based criteria (M, S&L, F, C&S and L&M) [2,6-8,10-12] were considered in pure torsion loading, where the normal stress amplitude $\sigma_a = 0$ and the shear stress amplitude τ_a corresponds to the fatigue resistance limit in shear t_{-1}. A modified version of C&S was also considered [14].

By deriving the dependence of the fatigue resistance limit from each criterion's equation, Matake and S&L present the fatigue resistance limit in shear t_{-1} as independent of a mean shear stress τ_m. This is a direct consequence of the fact that the critical plane is oriented in such a way that the normal stress acting on the critical plane is nil and the shear stress amplitude corresponds to the fatigue resistance limit in shear t_{-1}.

Following the same procedure, Findley eventually presents a dependence of the fatigue resistance limit on the mean shear stress. Once evaluated the critical plane's orientation, the criterion can be reduced to equation (1), where k and f are constants to the model.

$$\sqrt{\tau_a + k(\tau_a + \tau_m)} \leq f \tag{1}$$

At torsion fatigue limit, $\tau_a = t_{-1}$, yielding equation (2).

$$t_{-1} = -\frac{k^2 \tau_m}{(1+k^2)} + \sqrt{\frac{k^4 \tau_m^2}{(1+k^2)^2} - \frac{k^2 \tau_m^2 - f^2}{(1+k^2)}} \tag{2}$$

In regards to both C&S and modified C&S, in pure torsion loading the fracture plane is $\varphi_f = \pi/2$, meaning that the critical plane is given by $\varphi_c = \pi/2 - \delta$, where δ is exclusively dependant on material properties [1,2,13-15]. This reduces the stresses acting on the critical plane to $C_a = \tau_a \sin(2\delta)$, $N_a = \tau_a \cos(2\delta)$ and $N_m = \tau_m \cos(2\delta)$. By substituting such values in both C&S and modified C&S, one may encounter equations (3) and (4), where f_{-1} and t_{-1} are the fatigue resistance limits, respectively for bending and torsion loadings, τ_m is the mean shear stress and σ_u is the ultimate tensile stress.

$$t_{-1} = f_{-1} - \tau_m \tag{3}$$

$$t_{-1} = f_{-1}\left(1 - \frac{\tau_m}{\sigma_u}\right) \tag{4}$$

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As to the L&M criterion, it can also be applied to pure torsion loading yielding the same C_a, N_a and N_m evaluated in the C&S model. By determining δ [1] and by substituting $\tau_a = t_{-1}$ one finds that $t_{-1}/f_{-1} = 1/\sqrt{3}$, meaning that the fatigue resistance limit t_{-1} in torion is independent of τ_m, with the limitation of being a fixed fraction of the fatigue resistance limit f_{-1} for normal loadings.

Figure 1 demonstrates the dependence of the fatigue resistance limit on the mean shear stress according to C&S, Modified C&S and Findley. The curves were obtained for a hard steel considering $f_{-1} = 313.19\,MPa$, $t_{-1} = 196.2\,MPa$ and $\sigma_u = 704.1\,MPa$.

In an effort to evaluate the influence of τ_m on the applicability of the selected models, a number of experimental constant amplitude cyclic loading conditions encountered in the literature were considered [6,16]. Such loading conditions are relative to two alloy steels (42CrMo4 and 34Cr4) and they correspond to the fatigue limit state above which fatigue failure occurs and below which fatigue-life extends over a very high number of cycles (theoretically infinite-life). Tables 1 and 2 indicate the loading conditions as well as the material properties considered for each steel alloy.

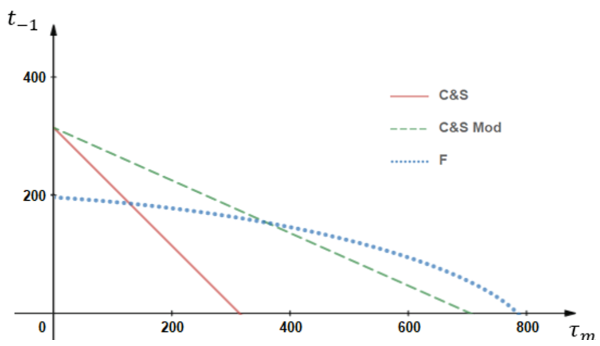

Figure 1. *Variation of the fatigue resistance limit in pure torsion,* t_{-1}*, with the mean shear stress* τ_m*.*

The error index can be defined as shown in equation (5). It will be used essentially to compare the left hand side of the criteria's main equation to its right hand side. The LHS is associated to the driving force to failure, i.e. it is associated to the loading conditions applied to the material. On the other hand, the LHS is related to the material's fatigue resistance limits.

$$I = \frac{LHS - RHS}{RHS} \times 100\% \qquad (5)$$

Since the given loading conditions are expected to lead the material to a critical condition where failure is on the threshold of taking place (in the order of one million cycles), error indices I tending to zero indicate that the criterion in question is in good agreement with the expected fatigue behaviour. In addition, positive I values are indicative of fatigue failure in a situation where failure is not observed, hence the criterion is considered to be conservative. On the other hand, negative I values indicate that the adopted criterion is non-conservative, as it may permit an increase in the applied loads, leading to higher risk of failure [17].

Results and discussion

The error index values were obtained for all seven loading conditions, evaluated according to each criteria (M, S&L, F, C&S, Mod C&S and L&M) including the mesoscopic scale-based criterion proposed by Papadopoulos (P) [6,18,19]. The latter is independent of critical plane evaluation, and its equation itself does not consider the influence of mean shear stress. All error indices are available on Figures 2 and 3 [20]. S&L may also be referred to as Modified Wöhler Curve Method (MWCM).

Matake yielded I values ranging from-9% to 22%, which is a range much wider than -6% to 8% normally encountered for combined fully reversed bend and torsion loadings where no mean shear stress is applied. This suggests that the inclusion of the mean shear in the model's equation indeed exerts a negative influence. It is important to observe that 5/7 error indices yielding from Matake are situated within -10% and 10% range, signalling fair predictive capability of the criterion in the presence of mean shear stress.

With I values ranging from 0% to 17%, the S&L criterion is shown to be conservative, with 4/7 of the I values within the ±10% range, indicating fairly good capability. As to Findley,

***Table 1**. Loading conditions applied for 42CrMo4, where f_{-1}=398 MPa, t_{-1}=260 MPa and σ_u=1025 MPa*

Loading condition	σ_a [MPa]	σ_m [MPa]	τ_a [MPa]	τ_m [MPa]	β [°]
1	266	0	128	128	0
2	283	0	136	136	90
3	333	0	160	160	180

***Table 2**. Loading conditions applied for 34Cr4, where f_{-1}=410 MPa, t_{-1}=256 MPa and σ_u=795 MPa*

Loading condition	σ_a [MPa]	σ_m [MPa]	τ_a [MPa]	τ_m [MPa]	β [°]
1	316	0	158	158	0
2	314	0	157	157	60
3	315	0	158	158	90
4	355	0	89	178	0

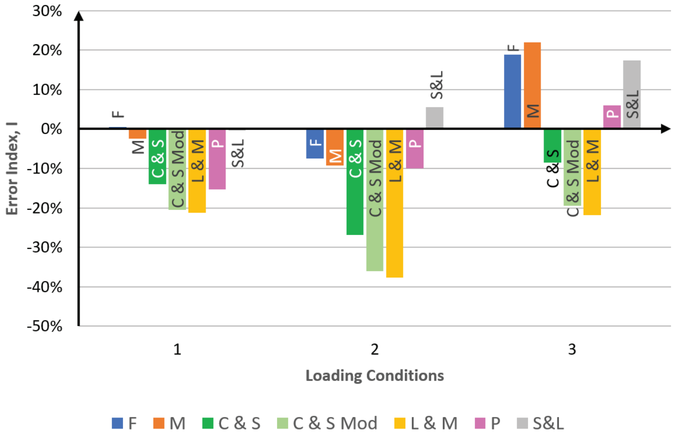

Figure 2. *Error indices I for 42CrMo4 steel alloy*

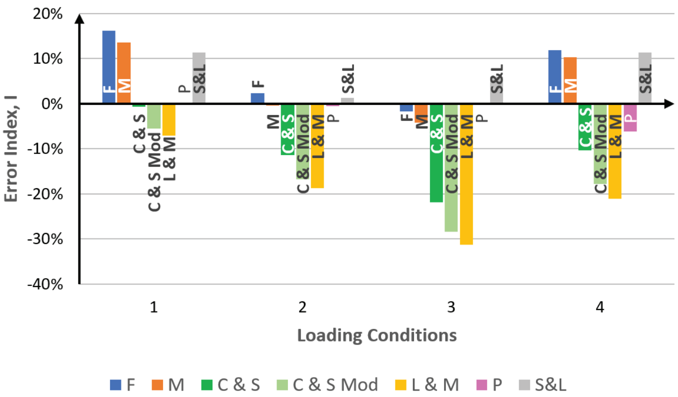

Figure 3. *Error indices I for 34Cr4 steel alloy*

I values range from -7% to 19%, which is a much broader range compared to the -1% to 11% reported for fully reversed bending and torsion loadings. Again, a negative influence on the model's predictive capability can be observed with the inclusion of the mean shear stress within the criterion. Findley, however, presented 5/7 of the error indices situated within the ±10% range, indicating fairly good capability.

As to C&S, modified C&S and L&M criteria, the I values obtained ranged from -38% to -1%, where for fully reversed bend and torsion loadings the I values were expected to range between -2% and 9% [6,17]. Thus, one can conclude that, except for four I values, the remaining 17/21 of the I values are significantly below -10%, indicating a fairly low predictive capability.

In regards to Papadopoulos, the I values ranged from -15% to 6%, indicating its predictive capability is far superior compared to the other criteria. For the 34Cr4 steel, the error indices are close to nil.

Conclusions

- Findley and C&S incorrectly predict a fatigue resistance limit in pure torsion dependant on the mean shear stress. According to L&M, the fatigue resistance limits in bend and torsion are restricted to a constant ratio $t_{-1}/f_{-1} = 1/\sqrt{3}$. For Matake and S&L, the fatigue resistance limit is independent of the mean shear stress.
- For combined bending and torsional loadings, the mean shear stress is one of the loading parameters and therefore is accounted for when the critical plane-based models are predicting fatigue behaviour.
- Papadopoulos' criterion, which does not depend on the mean shear stress, possesses the predictive capability superior to the others in the presence of mean shear stress.
- Therefore, it seems appropriate to propose that, in the presence of mean shear stress, multiaxial high cycle fatigue behaviour can be more safely evaluated by adopting mesoscopic scale-based criteria instead of the critical plane-based criteria considered in the present study.

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