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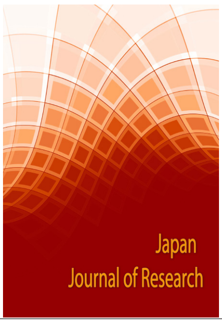

A hybridized geoid model for Ashanti Region, Ghana

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Abstract

GNSS positioning techniques deliver accurate three dimensional coordinates in Easting, Northing and geometric heights. However, the use of the geometric heights for topographic mapping is limited due to the unavailability of accurate National geoid models for their conversion into used Normal-Orthometric height systems in Ghana. Though global gravimetric geoids such as EGM2008 are readily available, these are not suitable for direct conversion of geometric heights from GNSS into the useful normal-orthometric height system used because local geoids are offset in an unpredictable way from such global geoids. In this investigation, a new hybrid geoid model for the Ashanti Region is developed and tested. The developed model incorporates the contribution of global geopotential models (GGMs) provided by EGM2008 together with GPS/Levelling using a remove and restore-like approach to develop a surface corrector model that separates the deterministic trend from the stochastic errors which then is separately modelled using the Gauss-Markov method. The main idea is thence to model the anomalies after comparing the gravimetric and quasi-geoid models from several collocated points. A Four-parameter Helmert transformation model have been applied to reduce data biases and provide a fitting quasi geoid heights obtained from the GGMs to those from GPS/levelling data. Whereas the gravimetric geoid alone has revealed that the vertical datum offset from the National height datum over Ashanti Region was at the level of 8cm up to 1.0m with a mean offset of 44cm. The Hybrid geoid model (ASHGeoid21) computed through the Remove-Compute-Restore (RCR) procedure resulted in geoid heights that are offset from Local geoid values at nineteen additional checkpoints for up to ± 50cm. with a standard deviation of ±10cm. It is recommended that, the hybrid residual height anomalies of a maximum of 50cm, could be further improved with the inclusion of more GPS/levelling points.

Introduction

Over the last few decades, the importance of the geoid has increased substantially due to the widespread use of Global Navigation Satellite Systems (GNSS) for positioning and navigation which additionally gives ellipsoidal heights. It is well known that, with the use of a suitable geoid model, Orthometric heights could be derived directly from the ellipsoidal heights.

The Geoid is the equal gravity potential surface of the earth which coincides with the mean sea level over the oceans if the ocean surface has equal density and is undisturbed by tides and currents. It is the reference surface used to measure precise elevations called orthometric heights of features i.e., the vertical distance H from the geoid surface along the plumbline to any point of interest is its orthometric height. However, the geoid surface is not the only one that can be used as a reference for measuring heights. An ellipsoid surface is at times used and in that case, the vertical distance from ellipsoid surface to places of interest are called ellipsoidal or geometric heights.

The physical realization of the geoid involves mean sea level determinations through hourly tide gauge observations over a period of 18.6 years to account for varying tides. However, in practice National determinations are done over shorter times and at a single point and are subject to varying ocean currents and salinity variations, and hence do not necessarily coincide with the geoid but may rather be quasi-geoids. This would rather make leveled National heights referenced to such quasi-geoids to instead result in Normal-orthometric heights.

Figure 1 shows four surfaces, namely the earth's topographic surface, the geoid surface, the quasi-geoid surface and the ellipsoid surface for purposes of height illustrations.

The Orthometric Height H is the plumbline distance from the geoid Po to the point of interest P. The Normal-Orthometric height H* is the distance from the quasi-geoid Q to the point of interest P. The geometric height h is the distance from the ellipsoid surface Qo to the point of interest P. The geoid height N is the difference between the ellipsoid and the geoid surfaces measured as the distance Qo to Po. Mathematically, $\approx h-H$, called the Geoid model is the distribution of N

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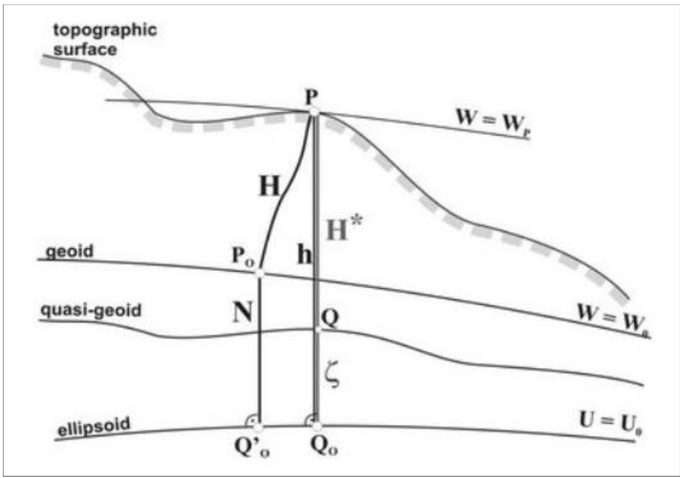

Figure 1. *Four Earth Surface Models and heights*

values that can be used to convert ellipsoidal heights to orthometric heights. The gravimetric Geoid model is computed directly using gravity measurements to obtain N values as the differences between the WGS84 equipotential ellipsoid surface and the geoid surface. A geometric Geoid model can equally be obtained from combining ellipsoidal heights obtained through GPS observations and their collocated levelled heights. The paradox however is that, the levelled heights may not be true orthometric heights H but rather normal-orthometric heights H* and hence the gravimetric geoid models would not model exactly the geometric geoid for direct conversion between ellipsoidal and National Reduced level heights.

Nonetheless, GNSS techniques for positioning yield three dimensional coordinates including the ellipsoidal heights readily, timely, cheaply, and under any weather conditions unlike the orthometric heights which are obtainable only through cumbersome traditional spirit levelling operations. Though the ellipsoidal heights cannot be used directly in surveying and engineering projects where orthometric heights are required, they can be converted into orthometric heights using a suitable geoid model. Gravimetric Geoids such as EGM2008 provides a geoid model that can be used in the conversion but its geoid heights do not match the precise point-wise local geoid undulations determined from GPS/leveling on benchmarks. This shows the global geoid as a height reference does not coincide with the local vertical datum or local geoid so the gravimetric geoid is not suitable for any direct conversion of ellipsoidal to orthometric heights without any modifications. On the other hand, geoid undulations determined by GPS/levelling method alone cannot be obtained to replace fully the gravimetric geoid because precise levelling is hard to be established over mountainous areas.

The problem can be resolved by developing a precise local geoid model through a hybridization approach which would require a correction model for the gravimetric geoid to account for the quasi-geoid to geoid dissimilarity since levelled heights are not referenced to the global geoid. To achieve a refined geoid model with high spatial resolution and accuracy, an available gravimetric geoid could thence be combined with GPS/leveling geoid to obtain a hybridized geoid [1-4].

Height Systems and Geoid Models

Height systems and height datums

A height system expresses the metric distance of some point on the surface of the earth from a reference surface along a well-defined path [5]. Thus different height coordinates can result for the same point dependent on the datum surface and the path. Accordingly, heights referenced to the geoid surface and measured along plumb lines are orthometric heights whereas those related to a referenced ellipsoid are ellipsoid heights, those referenced to the quasi-geoid surface are normal-orthometric heights [6,7].

The physical realization of both the Geoid and the Quasi-geoid are by tide gauge measurements. This is the methodology used to determine Mean Sea Levels (MSL). Since however, the MSL determined at a point is by no means global, the fit between the "global" MSL and the locally determined representations as geoids cannot be confirmed. Therefore, the geometry of physical references realized and used as the vertical datums (MSL) determined at different tide gauge positions infers that the equipotential surfaces passing through the differing reference tide gauges would only define local geoids which could well be offset by several meters from each other and also from the global average [1,8]. Furthermore, to correctly determine geoid, MSL observations should take into account the luni-solar tide, which has a full period of 18.6 years [6]. This is rarely possible for Height datum establishment which may rely on observations done only at one station and over a limited expanse of time. Adopting a single tide-gauge as the zero point for the realization of local vertical datums means there could be offsets among various vertical datums on a global scale [1] and this could pose a problem when integrating height data from different vertical datums.

The Ghanaian Height Datum (GHD) is actually a version of the normal-orthometric height system since the mean sea level determined could pass for a quasi-geoid and not a true geoid because no orthometric correction was applied and the observations were only for a limited time from 9th April 1922 to 30th April 1923 [9]. This implies that discrepancies could exist between gravimetric geoid heights and GPS-levelling geoid (geometric geoid) based on the GHD. Therefore, it is necessary to apply an additional transformation surface to account for these differences when warping the gravimetric geoid model to fit the GHD [9,10].

Geoid models

A geoid model is a mathematical representation of the geoid for a specific area which may be used to generate geoid separations for user's points in a network.

A gravimetric Geoid model that attempts to model the geoid as well as possible using gravity data but with no attempt to tie to an official vertical datum. Gravimetric Geoid Models are developed from surface gravity data, and terrain models. Gravimetric geoid modeling involves solving a Boundary Value Problem (BVP) to obtain the boundary value in the form of the geoid surface. This is achieved by applying methods such as Least Square Collocation (LSC), Least Square Modification (LSMS) of Stoke formula, or the use of spherical harmonic functions developed in series. Several EGM (Earth Gravitational Model) series have been computed and published for global use. They include the EGM96 developed by the NGA (National Geo-Spatial Intelligence Agency) in 1996 and the EGM2008 which used harmonics of degrees 2190 and order 2159 with a resolution of 5' [11].

Geometric geoids use a geometric approach involving a combination of GPS derived heights and levelling heights [12]. From GPS derived heights (h) and levelling heights (H) at benchmarks, geoid heights (N) are calculated. The simplified mathematical model of the relationship between geometric (h) and orthometric (H) heights showing their positional dependency is properly written as:

$$N(\varphi,\lambda) = h(\varphi,\lambda) - H(\varphi,\lambda) \quad (1)$$

The Hybrid Geoid Model is one that models the geoid but also ties it to an official vertical datum. It thence involves the integration of geometric and gravimetric geoid models. The input data for the hybrid geoid generation are GNSS/leveling geoid heights (N_{GNSS}) and a gravimetric geoid grid (N_{GRAV}). The calculation of offsets for each GNSS/leveling point is followed by the generation of corrector surface which then is applied to fit the obtained surface to the gravimetric geoid grid. A set of well distributed GNSS/leveling data is used to model the correction surface. The hybrid model depends on residuals between N_{GNSS} and N_{GRAV} calculated according equation:

$$\Delta N(\varphi,\lambda) = N_{GNSS}(\varphi,\lambda) - N_{GRAV}(\varphi,\lambda) \quad (2)$$

where ΔN(φ,λ) is to be modeled. The hybrid geoid model is obtained from the sum of the gravimetric geoid and the corrector.

$$N_{HYD}(\varphi,\lambda) = N_{GRAV}(\varphi,\lambda) + \Delta N(\varphi,\lambda) \quad (3)$$

Materials and methodology

The Ashanti Region of Ghana, covers latitude 5° 48′ North and 7° 36′N and longitude 0° 12′ West and longitude 2° 24′ West. It is located in the South Western middle belt of Ghana and is the third largest administrative region in Ghana. A total of 87 GPS/leveling stations were used (Figure 2). Their ellipsoidal heights are referenced to the WGS84 ellipsoid, and their leveled heights are above the Ghana National Height Datum of mean sea level. The step by step methodology adopted is illustrated in Figure 3.

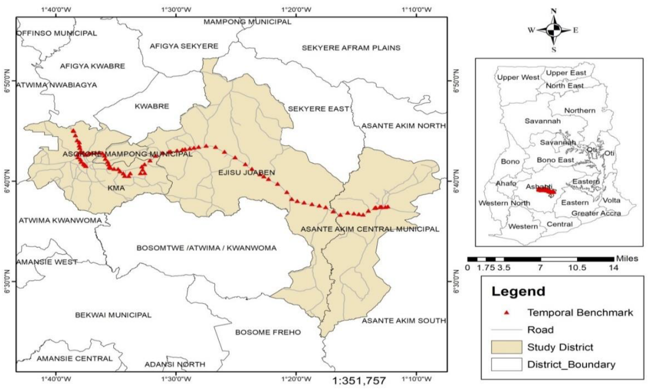

Figure 2. Map showing GPS/levelling Stations

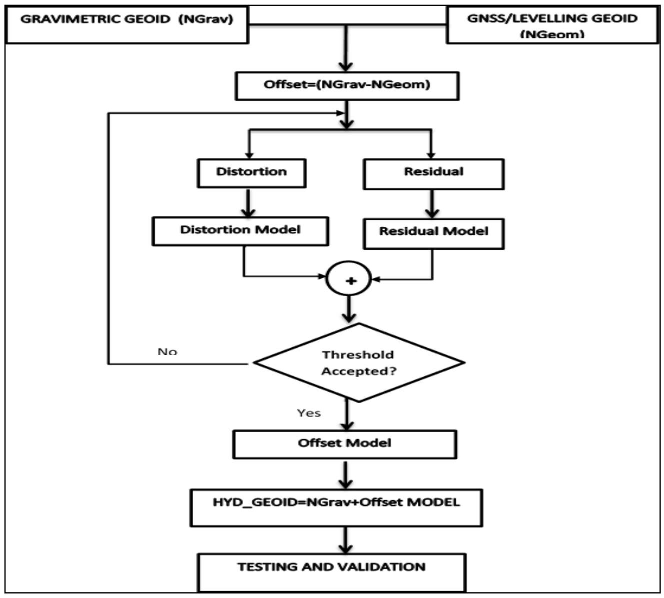

Figure 3. Flowchart of Methodology

Construction of EGM2008 extracted geoid model

The Gravimetric Geoid Heights from EGM2008 for the 87 GPS/Levelling stations are evaluated in Mat Lab to accuracy of 0.001m using function N = geoid_height (latitude, longitude, model name). Alternatively, a GeoidEval utility is used to compute geoid undulation using Earth Gravitational Model of 2008 (EGM2008) [13] to obtain the gravitational geoidal heights. Table 1 gives the summary statistics of geoid heights and geoid height difference (offsets)

Trend modelling

The deterministic transformation model that could be used to effectively fit the differences between the Gravimetric Geoid and the Geometric Geoid has to be determined empirically. Several modelling equations (4) to equation (9), are available for removing trend from the data using the offsets values obtained. In this study, the four parameter Helmert-type similarity transformation model given by $\Delta N = a_0 + a_1 \cos\varphi \cos\lambda + a_2 \cos\varphi \sin\lambda + a_3 \sin\varphi + \varepsilon$ is employed in order to remove distortions in these sets. The use of the model de-trends the data but leaves stochastic random residual errors which would be modelled using the least square collocation technique such as the Gauss-Markov model.

$$\Delta N = a_0 + a_1\varphi + a_2\lambda + \varepsilon \quad (4)\ (Linear\ Method)$$

$$\Delta N = a_0 + a_1\varphi + a_2\lambda + a_3\varphi^2 + a_4\varphi\lambda + a_5\lambda^2 + \varepsilon \quad (5)(2nd\ Order)$$

$$\Delta N = a_0 + a_1\varphi + a_2\lambda + a_3\varphi^2 + a_4\varphi\lambda + a_5\lambda^2 + a_6\varphi^3 + a_7\varphi^2\lambda + a_8\varphi\lambda^2 + a_9\lambda^3 + \varepsilon \quad (6)(3rd\ order)$$

$$\Delta N = a_0 + a_1 \cos\varphi \cos\lambda + a_2 \cos\varphi \sin\lambda + a_3 \sin\varphi + \varepsilon \quad (7)(4\text{-}Parameter\ Helmert)$$

$$\Delta N = a_0 + a_1 \cos\varphi i \cos\lambda i + a_2 \cos\varphi i \sin\lambda i + a_3 \sin\varphi i + a_4 \sin 2\varphi i + vi \quad (8)(5-Parameter)$$

$$\Delta N = a_0 + a_1 \cos\varphi i \cos\lambda i + a_2 \cos\varphi i \sin\lambda i + a_3 \sin\varphi i$$
$$+ a_4 \frac{\cos\varphi i \sin\varphi i \cos\lambda i}{\sqrt{1 - e2 \sin 2\varphi i}} + a_5 \frac{\cos\varphi i \sin\varphi i \sin\lambda i}{\sqrt{1 - e2 \sin 2\varphi i}} + a_6 \frac{\sin 2\varphi i}{\sqrt{1 - e2 \sin 2\varphi i}} + vi \quad (9)\ (7 - Parameter)$$

Modeling residuals

The offsets are decomposed into a distortion and the residual components. The distortion component which may contain the long and/or medium wavelength errors is removed by the trend model thus leaving the residual height anomalies. It is expected that these residuals would assume a stochastic form. Therefore, they could be interpolated to the same grid resolution as the gravimetric geoid by employing a Least-Squares Collocation method and a second-order Gauss Markov model is used for evaluating their covariance function. The empirical and fitted covariance functions of the residual height anomalies is required in the least square collocation technique from which we determine the best correlation length to use. After removing distortions from the offsets the remaining residual offsets ε is to be modelled using a second-order Gauss Markov model as .

$$K(\quad) = K(1 + A\quad)e \qquad (10)$$

where Ko is the variance of the observations. A is a parameter related to the correlation length; ρ is the distance measured in km. where the covariance falls to half the zero variance.

Correction surface model

The overall offset model is formed by adding residual components from the grid to the distortion component given by the trend surface model. This result is then added to the gravimetric geoid to obtain the hybrid geoid. Over 2 million grid points were extracted and used within the study area from EGM2008 from the International Centre for Global Earth Models (ICGEM platform) with a grid spacing of 0.1′ by 0.1′.

Results and Discussion

The Helmert Trend Surface model

The offset values modelled were obtained from the difference between the gravimetric and geometric geoids as:

$$\text{Offset } \Delta N = N_{Geom} - N_{Grav}$$

The results clearly indicate the non-coincidence of the gravimetric and geometric geoids and accentuates the fact that the local geoid is offset from the global gravimetric geoid. Such offset values would be made up of two components – a deterministic component that may be modelled mathematically using a trend surface model, and a stochastic component which may be modelled differently using the Gauss-Markov model. The trend surface model which yields the distortion component is given in (11)

$$\Delta N = -1.83817 + 1.03200\cos\varphi\cos\lambda - 0.10971\cos\varphi\sin\lambda + 3.11999\sin\varphi \qquad (11)$$

It is noted that, the offset is decomposed into the distortion trend component ΔN_1 given exactly by the trend equation, and then a stochastic component ΔN_2. The stochastic component ΔN_2 is modelled separately using the theory of random error distributions through the Gauss-Markov to produce a stochastic surface from which the stochastic error component for each point may be extracted.

The Gravimetric Geoid Surface for study area.

The gravimetric geoid surface extracted from EGM2008 covering the study area as shown in figure 4 together with a sample of geoidal height values for the surface. The gravimetric geoid is used as input together with the four parameter Helmert deterministic model to obtain the distortion components of corrections to be applied to the gravimetric geoid in order to obtain the hybrid geoid.

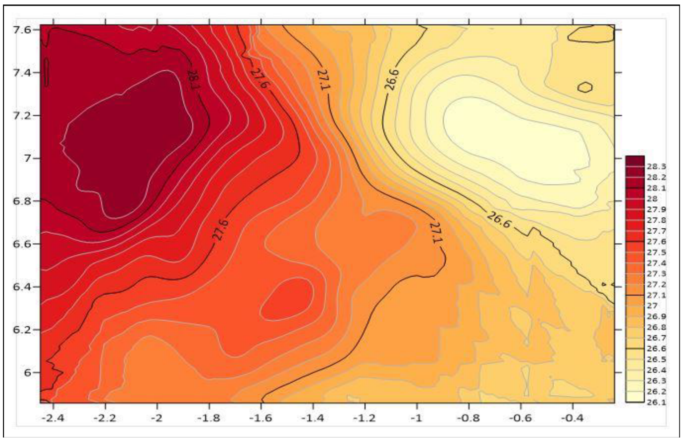

Figure 4. *Gravimetric grid surface extracted from EGM2008*

Deterministic surface model

The deterministic surface model component is obtained using the Helmert 4 parameter trend surface equation with the Gravimetric geoid values as input. This computes the trend surface value at each latitude and longitude position. These values are also used in producing the trend surface map shown in figure 5

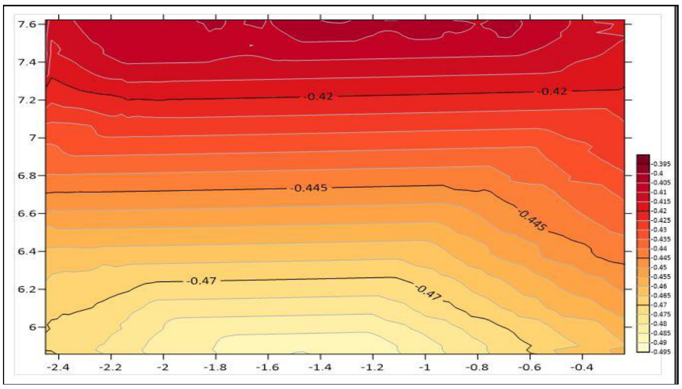

Figure 5. *Trend surface Correction model*

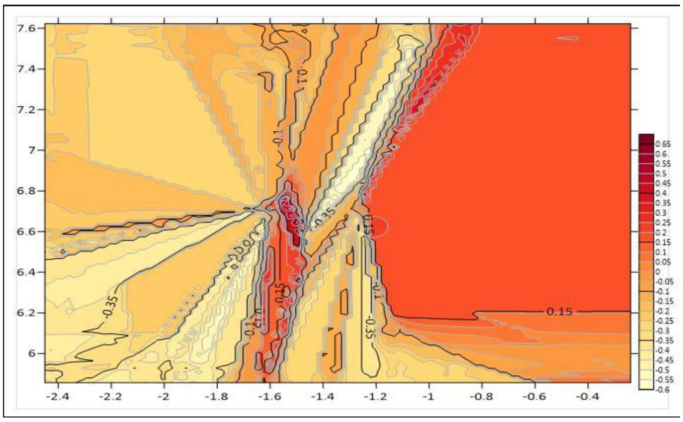

Figure 6. *Stochastic surface model*

Stochastic surface model

The residual components (ΔN_2) of the offset which is stochastic in nature is modeled using Gauss-Markov second order function to cover the study area. The stochastically modeled values are used to generate the stochastic surface shown in figure 6.

The total offset is the sum of the distortion trend component and the stochastic parts. That is:

$$\text{Offset} = \Delta N_1 + \Delta N_2 \qquad (12)$$

Figure 7 shows the combined offset surface obtained from adding the distortion model and the residual model components

The Hybrid Geoid model

The hybrid geoid model shown as surface in Figure 8. is obtained from

$$N_{HYBRID} = N_{(Grav)} + \Delta N_1 + \Delta N_2 \qquad (13)$$

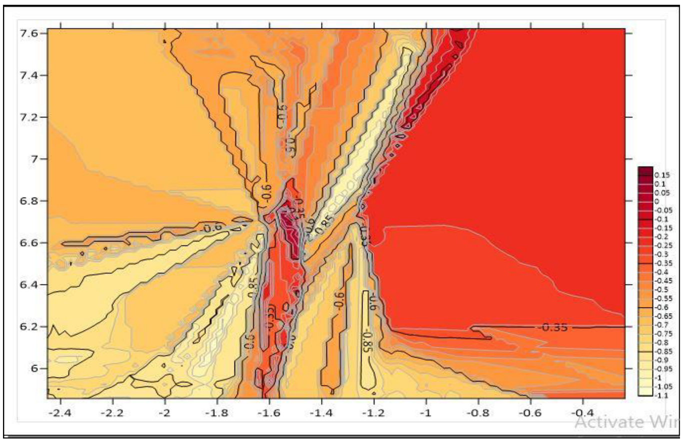

Figure 7. *Offset surface model*

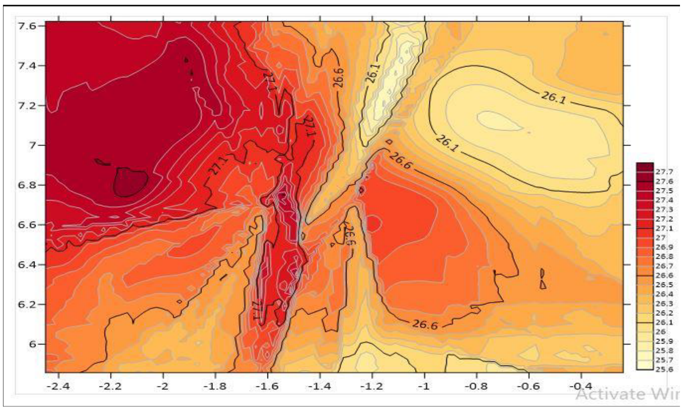

Figure 8. *Hybrid surface model*

The method of integration approach employed has taken into account the general trend of the separation between the gravimetric geoid and observed local geoid heights. The tilts of the gravimetric geoid model with respect to GPS/levelling data is removed using the least squares fitting of the four-parameter Helmet transformation. This removes the trend and the resulting residual stochastic component is modelled through a covariance stochastic function. The trend function itself has been modelled through an empirical method therefore could be improved with more GPS/levelling data to get a meaningful and reliable covariance function. The suitability or otherwise in terms of performance of the hybrid model has to be determined through validation.

The Hybrid Geoid model

Nineteen additional GPS/levelling stations not used for the model creation were used as checkpoints for validating the hybrid geoid developed. The Validation N values for the Checkpoints as compared with the Hybrid Geoid N values shows greatest absolute difference of 49.7cm and a standard deviation of 10cm which is just some 2.4cm standard error of the mean.

The resulting model thence show substantial improvement in Geoid accuracies which can be used to obtain orthometric heights from ellipsoidal height values up to accuracies of 10cm and standard error in mean of 2.4cm. The developed hybrid model for Ashanti Region, called ASHGeoid21, incorporated the contribution of global geopotential models (GGMs) provided by EGM2008 using a remove and restore-like approach and developed a surface correction model. The improvement is significant as it has enabled an achievement of centimeter accuracy for heights from GNSS derived heights. The absolute agreement of the transformed geoid with GPS/leveling data is 22mm at 17checkpoints. The final Hybrid height anomalies were computed on a 0.1' × 0.1' grid and it is found that our hybrid geoid model ASHGeoid21 has improved the geoid heights by about 50% on a grid scale with other geoid undulations derived from EGM2008 and 59% on a point-wise scale when using GPS/levelling data. Further improvement with inclusion of more GPS/Levelling data in the North-South direction is recommended in future works.

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